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Key Curriculum Press
1150 65th Street
Emeryville, CA 94608
(510) 595-7000
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www.keypress.com

Printed in the United States of America
10 9 8 7 6 5 4 3 2 1 13 12 11 10 09
ISBN 978-1-60440-008-3
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Introduction

The authors of Discovering Advanced Algebra: An Investigative Approach are aware of the importance of students developing algebra skills along with acquiring concepts through investigation. The student book includes many skill-based exercises. These More Practice Your Skills worksheets provide problems similar to the Practice Your Skills exercises in Discovering Advanced Algebra. Like the Practice Your Skills exercises, these worksheets allow students to practice and reinforce the important procedures and skills developed in the lessons. Some of these problems provide non-contextual skills practice. Others give students an opportunity to apply skills in fairly simple, straightforward contexts. Some are more complex problems that are broken down into small steps, and some have several parts, each giving practice with the same skill.

You might assign the More Practice Your Skills worksheet for every lesson, or only for those lessons your students find particularly difficult. Or you may wish to assign the worksheets on an individual basis, only to those students who need extra help. One worksheet has been provided for every lesson. The worksheets for Chapter 0 provide a review of beginning algebra skills. To save you the time and expense of copying pages, you can give students the inexpensive More Practice Your Skills Student Workbook, which does not have answers. These worksheets are also available to students at www.keymath.com. Though the limited reproduction permission allows you to copy pages from More Practice Your Skills with Answers for use with your students, the consumable More Practice Your Skills Student Workbook should not be copied. You can also use the Discovering Advanced Algebra TestCheck™: Test Generator and Worksheet Builder™ CD to generate additional practice sheets for students who need further practice.
Lesson 0.1 • Pictures, Graphs, and Diagrams

1. Find the slope of each line.
   a.  
   b.  
   c.  

2. Solve.
   a. \( \frac{4}{7} = \frac{a}{21} \)  
   b. \( \frac{5}{9} = \frac{20}{b} \)  
   c. \( \frac{c}{42} = \frac{11}{14} \)  
   d. \( \frac{54}{d} = \frac{9}{13} \)  
   e. \( \frac{w}{8} = \frac{81}{72} \)  
   f. \( \frac{3}{14} = \frac{60}{x} \)

3. Find the slope of the line that passes through each pair of points.
   a. (0, 1) and (3, 8)  
   b. (3, 5) and (5, 0)  
   c. (−2, 3) and (4, 9)  
   d. (−8, 0) and (0, −8)  
   e. (−5, −2) and (−8, 4)  
   f. (−1, 7) and (−5, 2)

4. Convert each decimal value to fraction form. Write all fractions in lowest terms.
   a. 0.875  
   b. 1.3  
   c. 0.5  
   d. 0.4\(\overline{16}\)  
   e. 1.75  
   f. 0.18
Lesson 0.2 • Symbolic Representation

1. Explain what you would do to change the first equation to the second.
   a. \( x + 7 = 22 \)  \( x = 15 \)
   b. \( 8y = 72 \)  \( y = 9 \)
   c. \( \frac{w}{-11} = 2.5 \)  \( w = -27.5 \)

2. Solve.
   a. \( 12 + a = 39 \)
   b. \( 42 - b = 33 \)
   c. \( 25c = 375 \)
   d. \( 5 + 3d = -7 \)
   e. \( -14 + 3p = -9p - 21 \)
   f. \( 15 - 6q = 2q + 9 \)

3. Rewrite each expression without parentheses.
   a. \( -3(7 - y) \)
   b. \( -12q(12 - q) \)
   c. \( -7y(y^2 - 3y) \)
   d. \( (2r - 5)(3r) \)
   e. \( (-8s + 5)(-6s) \)
   f. \( 8z(2z^2 - 15) \)

4. Substitute the given value of the variable(s) in each expression and evaluate.
   a. \( 5(y + 7) \) when \( y = -12 \)
   b. \( -2a + 5b \) when \( a = -3 \) and \( b = 6 \)
   c. \( 0.2a - 0.4b + 0.6c \) when \( a = 20 \), \( b = -32 \), and \( c = 16 \)
   d. \( \frac{3}{8}f - \frac{5}{11}h + \frac{9}{7}j \) when \( f = -8 \), \( h = -22 \), and \( j = 21 \)
Lesson 0.3 • Organizing Information

1. Use the distributive property to expand each expression, and combine like terms when possible.
   a. $2.6(w - 4)$
   b. $4.3 - (2y + 8.9)$
   c. $6.8s - 2.8(t - 3)$
   d. $-2(6u + 8) + 5(3u - 2)$
   e. $\frac{8}{11}(22z + 33) - \frac{12}{13}(26z + 13)$

2. Solve.
   a. $6(p - 9) = -33$
   b. $-8(q + 12) = 10.4$
   c. $22 - 5(s + 6) = 32$
   d. $4.5(z + 6) + 24.5 = -7$

3. Rewrite each expression using the properties of exponents so that the variable appears only once.
   a. $(m^4)(m^8)$
   b. $\frac{(p^{13})(p)}{p^{24}}$
   c. $(-3a^2b)^4$
   d. $\frac{(4x^5)(-5x)^2}{(10x)(-x^2)^3}$

4. Expand each product and combine like terms.
   a. $(x + 4)(x + 5)$
   b. $(2m - 3)^2$
   c. $(7p - 9)(7p + 9)$

5. Jerome can complete 13 homework problems in 52 minutes. The relationship between $y$, the number of homework problems he completes, and $x$, the time in minutes, is a direct variation.
   a. Find $k$, the constant of variation. Then, write the direct variation equation that relates $x$ and $y$.
   b. If Jerome continues to work at the same rate, how many problems can he complete in 2 hours?
Lesson 1.1 • Recursively Defined Sequences

1. Find the common difference, \(d\), for each arithmetic sequence and the common ratio, \(r\), for each geometric sequence.
   
   a. 1.5, 1.0, 0.5, 0, −0.5, . . .  
   b. 0.0625, 0.125, 0.25, . . .  
   c. −1, 0.2, −0.04, 0.008, . . .

2. Write the first six terms of each sequence and identify each sequence as arithmetic or geometric.
   
   a. \(u_1 = −18\)  
      \(u_n = u_{n−1} + 6\) where \(n \geq 2\)  
   b. \(u_1 = 0.5\)  
      \(u_n = 3u_{n−1}\) where \(n \geq 2\)

3. Write a recursive formula to generate each sequence. Then find the indicated term.
   
   a. 17.25, 14.94, 12.63, 10.32, . . . Find the 15th term.
   b. −2, 4, −8, 16, . . . Find the 15th term.

4. Indicate whether each situation could be represented by an arithmetic sequence or a geometric sequence. Give the value of the common difference, \(d\), for each arithmetic sequence and of the common ratio, \(r\), for each geometric sequence.
   
   a. Phil rented an apartment for $850 a month. Each time he renewed his annual lease over the next 3 years, his landlord raised the rent by $50.
   b. Leora was hired as a first-year teacher at an annual salary of $30,000. She received an annual salary increase of 5% for each of the next 4 years.

5. Write a recursive formula for the sequence graphed at right. Find the 42nd term.

\[ u_n = \begin{cases} 0 & \text{if } n = 1 \\ u_{n−1} + 10 & \text{if } n > 1 \end{cases} \]
Lesson 1.2 • Modeling Growth and Decay

1. Find the common ratio for each sequence and identify the sequence as growth or decay. Give the percent increase or decrease for each.
   a. 42, 126, 378, 1134, . . .
   b. 19.2, 3.84, 0.768, 0.1536, . . .
   c. 90, 99, 108.9, 119.79, . . .
   d. 1800, 1080, 648, 388.8, . . .

2. Write a recursive formula for each sequence in Exercise 1. Use $u_0$ for the first term given and find $u_5$.

3. Factor each expression so that the variable appears only once. For example, $x + 0.05x$ factors into $x(1 + 0.05)$.
   a. $y - 0.19y$
   b. $2A - 0.33A$
   c. $u_{n-1} - 0.72u_{n-1}$
   d. $3u_{n-1} - 0.5u_{n-1}$

4. Write a recursive formula for the sequence $3, -8.5, 26, -77.5, . . .$

5. Match each recursive formula to a graph.
   a. $u_0 = 35$
      $u_n = (1 - 0.3) \cdot u_{n-1}$ where $n \geq 1$
   b. $u_0 = 35$
      $u_n = (1 - 0.5) \cdot u_{n-1}$ where $n \geq 1$
   c. $u_0 = 35$
      $u_n = -0.5 + u_{n-1}$ where $n \geq 1$
   A. $u_n$
   B. $u_n$
   C. $u_n$
Lesson 1.3 • A First Look at Limits

1. For each sequence, find the value of \( u_1, u_2, \) and \( u_3 \). Identify the type of sequence (arithmetic, geometric, or shifted geometric) and tell whether it is increasing or decreasing.
   a. \( u_0 = 25 \)
      \[ u_n = u_{n-1} + 8 \quad \text{where} \quad n \geq 1 \]
   b. \( u_0 = 10 \)
      \[ u_n = 0.1u_{n-1} \quad \text{where} \quad n \geq 1 \]
   c. \( u_0 = 48 \)
      \[ u_n = u_{n-1} - 6.9 \quad \text{where} \quad n \geq 1 \]
   d. \( u_0 = 500 \)
      \[ u_n = (1 - 0.80)u_{n-1} + 25 \quad \text{where} \quad n \geq 1 \]

2. Solve.
   a. \( r = 0.9r + 30 \)
   b. \( s = 25 + 0.75s \)
   c. \( t = 0.82t \)
   d. \( v = 45 + v \)
   e. \( w = 0.60w - 20 \)
   f. \( z = 0.125z + 49 \)

3. Find the long-run value for each sequence.
   a. \( u_0 = 48 \)
      \[ u_n = 0.75u_{n-1} + 25 \quad \text{where} \quad n \geq 1 \]
   b. \( u_0 = 12 \)
      \[ u_n = 0.9u_{n-1} + 2 \quad \text{where} \quad n \geq 1 \]
   c. \( u_0 = 62 \)
      \[ u_n = (1 - 0.2)u_{n-1} \quad \text{where} \quad n \geq 1 \]
   d. \( u_0 = 45 \)
      \[ u_n = (1 - 0.05)u_{n-1} + 5 \quad \text{where} \quad n \geq 1 \]

4. Write a recursive formula for each sequence. Use \( u_0 \) for the first term given.
   a. 0, 20, 36, 48.8, . . .
   b. 100, 160, 226, 298.6, . . .
   c. 50, 36, 27.6, 22.56, . . .
   d. 40, 44, 50.4, 60.64, . . .
Lesson 1.4 • Graphing Sequences

1. Write five ordered pairs that represent points on the graph of each sequence.
   a. \( b_0 = 2 \)
      \( b_n = b_{n-1} + 8 \) where \( n \geq 1 \)
   b. \( b_0 = 10 \)
      \( b_n = 0.1b_{n-1} \) where \( n \geq 1 \)
   c. \( b_0 = 0 \)
      \( b_n = 2.5b_{n-1} + 10 \) where \( n \geq 1 \)
   d. \( b_0 = 150 \)
      \( b_n = 0.8b_{n-1} - 10 \) where \( n \geq 1 \)

2. Match each formula with a graph and identify the sequence as arithmetic or geometric.
   a. \( u_0 = 10 \)
      \( u_n = 1.5u_{n-1} \) where \( n \geq 1 \)
   b. \( u_0 = 30 \)
      \( u_n = u_{n-1} + 5 \) where \( n \geq 1 \)
   c. \( u_0 = 80 \)
      \( u_n = 0.75u_{n-1} \) where \( n \geq 1 \)
   A. \[ u_n \]
   B. \[ u_n \]
   C. \[ u_n \]

3. Imagine the graphs of the sequences generated by these recursive formulas. Describe each graph using exactly three of these terms: arithmetic, geometric, shifted geometric, linear, nonlinear, increasing, decreasing.
   a. \( t_0 = 50 \)
      \( t_n = t_{n-1} - 10 \) where \( n \geq 1 \)
   b. \( a_0 = 1000 \)
      \( a_n = 0.7a_{n-1} + 100 \) where \( n \geq 1 \)
   c. \( u_0 = 35 \)
      \( u_n = u_{n-1} \cdot 1.75 \) where \( n \geq 1 \)
   d. \( t_0 = 150 \)
      \( t_n = (1 - 0.15)t_{n-1} \) where \( n \geq 1 \)
Lesson 1.5 • Loans and Investments

1. Assume that each of the sequences below represents a financial situation. Indicate whether each represents a loan or an investment, and give the principal and the deposit or payment amount.

   a. \( a_0 = 1000 \)
      \[ a_n = (1 + 0.04)a_{n-1} + 100 \quad \text{where} \quad n \geq 1 \]

   b. \( a_0 = 130,000 \)
      \[ a_n = \left(1 + \frac{0.0625}{4}\right)a_{n-1} - 1055 \quad \text{where} \quad n \geq 1 \]

   c. \( a_0 = 1825 \)
      \[ a_n = \left(1 + \frac{0.075}{12}\right)a_{n-1} + 120 \quad \text{where} \quad n \geq 11 \]

2. For each financial situation represented by a sequence in Exercise 1, give the annual interest rate and the frequency with which interest is compounded.

3. Find the first month’s interest on each loan.

   a. $20,000 loan; 6% annual interest rate

   b. $1,650 loan; 4.6% annual interest rate

4. Write a recursive formula for each financial situation.

   a. You take out a home mortgage for $144,500 at 6.2%, compounded monthly, and make monthly payments of $990.

   b. You enroll in an investment plan through your job that deducts $225 from your monthly paycheck and deposits it into an account with an annual interest rate of 3.75%, compounded monthly.
Lesson 2.1 • Box Plots

1. Find the mean, median, and mode for each data set.
   a. {2, 3, 5, 5, 7, 7, 8, 9, 10}       b. {210, 180, 188, 162, 170}
   c. {4.5, 20.7, 35.2, 28.8, 36.5, 40.5}       d. {5.3, 8.4, 5.3, 9.2, 10.6, 9.2}

2. Suppose that you have a data set containing 1000 test scores. How many scores would you expect to find matching each description?
   a. Above the median
   b. Below the first quartile
   c. Between the first and third quartiles
   d. Above the third quartile
   e. Below the third quartile
   f. Above the first quartile

3. Give the five-number summary for each data set.
   a. {10, 8, 6, 4, 2}
   b. {0, 30, 45, 50, 75, 80, 95}
   c. {8, 6, 8, 2, 9, 4, 4, 3, 1}
   d. {32, 55, 16, 70, 65, 55, 40, 49}

4. For each data set, find the median, the range, and the IQR.
   a. {18, 13, 15, 24, 20}
   b. {356, 211, 867, 779, 101, 543}

5. Match each box plot to one of the data sets below.
   a. [29, 16, 20, 28, 5, 50, 15]
   b. [30, 18, 22, 28, 31, 15, 50]
   c. [21, 12, 33, 44, 26, 15, 36]
   d. [48, 41, 35, 12, 15, 19, 26]
Lesson 2.2 • Measures of Spread

Name ___________________________  Period _________  Date ________________

1. For each data set, find the mean, the deviation from the mean for each value, and the standard deviation of the data set. (Round to the nearest tenth.)
   a. {12.4, 26.3, 9.8, 33.9, 7.6}  
   b. {235, 413, 505, 111, 700, 626, 357}

2. For each data set, give the mean and the standard deviation. Include appropriate units in your answers.
   a. The heights in inches of eight children are 32, 45, 39, 51, 28, 54, 37, and 42.
   b. The lengths in centimeters of six pencils are 8.5, 19.0, 11.8, 13.2, 16.4, and 6.1.
   c. The prices of seven music CDs are $13.50, $10.95, $9.95, $16.00, $12.50, $15.95, and $17.75.

3. Invent a data set with five data values such that the mean is 12 and the standard deviation is 3.

4. Identify all outliers in each data set using the $1.5 \cdot \text{IQR}$ definition.
   a. {20, 8, 32, 18, 105, 4, 45}  
   b. {3.2, 4.9, 1.6, 2.8, 5.5}  
   c. {35, 38, 5, 46, 49, 41, 52, 95}
Lesson 2.3 • Histograms and Percentile Ranks

1. The following data represent the ages of family members attending a family reunion.

   \{9, 5, 25, 29, 40, 48, 63, 56, 3, 32, 38, 53, 79, 0, 85, 87, 12, 14, 32, 5, 54, 67, 78, 75\}

   a. Find the percentile rank of the family member who is 14 years old.

   b. Draw a histogram for these data with 9 bins.

2. For each of the following histograms, give the bin width and the number of values in the data set. Then identify the bin that contains the median of the data.

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<td>210</td>
<td>363</td>
</tr>
<tr>
<td>329</td>
<td>329</td>
</tr>
</tbody>
</table>

3. Find each percentile rank.

   a. 460 out of 1000 students scored at least 30 points out of 50 on a standardized test. Find the percentile rank of a student who scored 30 points on the test.

   b. 76 out of 200 people living alone spend $800 a month or more on rent. Find the percentile rank of a person who spends $800 a month on rent.
Lesson 3.1 • Linear Equations and Arithmetic Sequences

Name ____________________________  Period _________  Date ____________

1. Find an explicit formula for each recursively defined arithmetic sequence.
   a. \( u_0 = 18.25 \)
      \( u_n = u_{n-1} - 4.75 \) where \( n \geq 1 \)
   b. \( t_0 = 0 \)
      \( t_n = t_{n-1} + 100 \) where \( n \geq 1 \)

2. Refer to the graph of the sequence.
   a. Write a recursive formula for the sequence. What is the common difference? What is the value of \( u_0 \)?

   b. What is the slope of the line through the points? What is the \( y \)-intercept?

   c. Write an equation for the line that contains these points.

3. Find the slope of each line.
   a. \( y = 5 + 3x \)
   b. \( y = 10 - x \)
   c. \( y = 0.6x - 0.8 \)

4. Write an equation in the form \( y = a + bx \) for each line.
   a. The line that passes through the points of an arithmetic sequence
      with \( u_0 = 11 \) and a common difference of 9

   b. The line that passes through the points of an arithmetic sequence
      with \( u_0 = -7.5 \) and a common difference of \(-12.5 \)
Lesson 3.2 • Revisiting Slope

1. Find the slope of the line containing each pair of points.
   a. (2, 6) and (4, 12)  
   b. (0, 7) and (5, 0)  
   c. \((\frac{1}{3}, \frac{2}{3})\) and \((\frac{5}{6}, -\frac{1}{6})\)

2. Find the slope of each line.
   a. \(y = 1.6 - 2.5x\)  
   b. \(y = -4(x - 7) + 12\)  
   c. \(y = 14.5 - 0.3(x - 30)\)

3. Solve.
   a. \(y = 6 - 2x\) for \(y\) if \(x = -4\)  
   b. \(y = a - 0.4x\) for \(a\) if \(x = 600\) and \(y = 150\)

4. Find the equations of both lines in each graph.
   a. 
   b. 
   c. What do the equations in part a have in common? What do you notice about their graphs?
   d. What do the equations in part b have in common? What do you notice about their graphs?
Lesson 3.3 • Fitting a Line to Data

Name ___________________________________________  Period __________  Date ________________

1. Write an equation in point-slope form for each line.
   a. 
      \[
      y - 3 = \frac{2}{3}(x - 1)
      \]
   b. 
      \[
      y - 2 = -\frac{3}{2}(x + 3)
      \]

2. Write an equation in point-slope form for each line.
   a. Slope 0.75 and passing through \((-4, 10)\)
   b. Parallel to \(y = 7 - 4x\) and passing through \((2, -5)\)

3. Solve.
   a. \(d = 9 - 4(t + 5)\) for \(d\) if \(t = 20\).
   b. \(y = 500 - 20(x - 5)\) for \(x\) if \(y = 240\).
   c. \(a_n = -3.5 + 0.4(n - 12)\) for \(n\) if \(a_n = 2.9\).

4. For each graph, use your ruler to draw a line of fit. Explain how your line satisfies the guidelines on page 138 of your book.
   a. 
      \[
      \]
   b. 
      \[
      \]
1. How should you divide the following sets into three groups for the median-median line?
   a. Set of 33 elements  
   b. Set of 64 elements  
   c. Set of 57 elements

2. Find the point with coordinates \((\text{median } x, \text{median } y)\) for each group of points.
   a. \((3, 4), (5, 8), (11, 9), (13, 10)\)
   b. \((0, 3), (2, 6), (3, 4), (5, 1), (7, 5)\)
   c. \((14, 20), (11, 11), (17, 13), (15, 19), (16, 22), (20, 18)\)
   d. \((2.5, 5.0), (4.1, 3.8), (1.6, 7.5), (5.9, 2.6)\)

3. Find an equation in point-slope form for the line passing through each pair of points.
   a. \((5, 8)\) and \((8, 2)\)  
   b. \((-1, 6)\) and \((9, -4)\)
   c. \((20, -14)\) and \((-30, 16)\)  
   d. \((44.2, -22.8)\) and \((25.2, 34.2)\)

4. Find an equation for each line described. Write your answer in the same form as the given line or lines.
   a. Line one-third of the way from \(y = 2x + 6\) to \(y = 2x + 15\)
   b. Line one-third of the way from \(y = 0.8x + 12.6\) to the point \((9, 48)\)
Lesson 3.5 • Prediction and Accuracy

1. Determine whether the given point lies above or below the given line.
   a. \( y = -2x + 6; (3, 1) \)
   b. \( y = 3.6x - 18.8; (10, 16.9) \)

2. The equation \( \hat{y} = 4x - 5 \) represents the median-median line for a set of data. The table at right gives the \( x \)-value and the residual for each data point. Find the \( y \)-value for each data point.

<table>
<thead>
<tr>
<th>( x )-value</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual</td>
<td>1</td>
<td>-1</td>
<td>2</td>
<td>-3</td>
</tr>
</tbody>
</table>

3. This table gives the number of students enrolled in U.S. public schools for various years.
   a. Find the median-median line for the data.
      Round all answers to one decimal place. Does the \( y \)-intercept make sense for the data?

<table>
<thead>
<tr>
<th>School year</th>
<th>Public school enrollment</th>
</tr>
</thead>
<tbody>
<tr>
<td>1909–10</td>
<td>17,814</td>
</tr>
<tr>
<td>1919–20</td>
<td>21,578</td>
</tr>
<tr>
<td>1929–30</td>
<td>25,678</td>
</tr>
<tr>
<td>1939–40</td>
<td>25,434</td>
</tr>
<tr>
<td>1949–50</td>
<td>25,111</td>
</tr>
<tr>
<td>1959–60</td>
<td>35,182</td>
</tr>
<tr>
<td>1969–70</td>
<td>45,550</td>
</tr>
<tr>
<td>1979–80</td>
<td>41,651</td>
</tr>
<tr>
<td>1989–90</td>
<td>40,543</td>
</tr>
<tr>
<td>1999–2000</td>
<td>46,812</td>
</tr>
</tbody>
</table>

(b) The World Almanac and Book of Facts 2007

b. Calculate the residuals.

c. Calculate the root mean square error for the median-median line.

d. What is the real-world meaning of the root mean square error?

e. The World Almanac predicts that the public school enrollment in the 2013–14 school year will be 49,737 students. Use your median-median line to predict enrollment in 2013–14 and calculate the residual of the Almanac’s prediction.

4. The list of numbers 3, \(-2\), 1, \(0\), \(-3\), \(-2\), 4 represents the residuals for a data set. Find the root mean square error for the set of residuals.
   (Round your answers to the nearest hundredth.)
Lesson 3.6 • Linear Systems

1. Identify the point of intersection listed below each system of linear equations that is the solution of that system.
   a. \[
   \begin{align*}
   2x + 5y &= 10 \\
   x - 3y &= -6
   \end{align*}
   \]
   \( (5, 0); (0, 2); (3, 1) \)
   b. \[
   \begin{align*}
   4x + 3y &= 4 \\
   3x - 2y &= -14
   \end{align*}
   \]
   \( (-2, 4); (0, \frac{4}{3}); (0, 7) \)
   c. \[
   \begin{align*}
   6x - 5y &= 0 \\
   x - y &= -1
   \end{align*}
   \]
   \( (0, 0); (-5, -6); (5, 6) \)

2. Write a system of linear equations that has each ordered pair as its solution.
   a. \((5, 4)\)
   b. \((-3, 8)\)
   c. \((3, 10.5)\)

3. Write an equation for each line described.
   a. Perpendicular to \(y = 2x - 3\) and passing through the point \((5, -4)\)
   b. Perpendicular to \(y = 1.5 + 0.25x\) and passing through the point \((5, -2)\)

4. Solve.
   a. \(8 - 3(x - 2) = 5 + 6x\)
   b. \(3.8t - 16.2 = 12 + 2.8(t + 3)\)

5. Use substitution to find the point \((x, y)\) where each pair of lines intersect. Use a graph or table to verify your answer.
   a. \[
   \begin{align*}
   y &= 3 - 2x \\
   y &= 5 + 2x
   \end{align*}
   \]
   b. \[
   \begin{align*}
   y &= 0.45x - 2 \\
   y &= -0.45x + 2
   \end{align*}
   \]
   c. \[
   \begin{align*}
   y &= 9 + 4(x - 3) \\
   y &= 15 - 2x
   \end{align*}
   \]
Lesson 3.7 • Substitution and Elimination

1. Solve each equation for the specified variable.
   a. \( r - s = 20, \) for \( s \)
   b. \( 5x - 8y = -10, \) for \( x \)
   c. \( 0.2m - 0.5n = 1, \) for \( n \)
   d. \( 250x + 400y = -50, \) for \( y \)

2. Graph each system and find an approximate solution. Then choose a method and find the exact solution. List each solution as an ordered pair.
   a. \[
   \begin{align*}
   x + y &= 1 \\
   2x - 2y &= 1
   \end{align*}
   \]
   b. \[
   \begin{align*}
   3x - 2y &= 6 \\
   -2x + 3y &= 0
   \end{align*}
   \]
   c. \[
   \begin{align*}
   5x + 4y &= 16 \\
   4x - 3y &= 12
   \end{align*}
   \]

3. Solve each system of equations.
   a. \[
   \begin{align*}
   3x - 4y &= 8 \\
   y &= x - 1
   \end{align*}
   \]
   b. \[
   \begin{align*}
   5x - 8y &= 8 \\
   -10x + 4y &= -7
   \end{align*}
   \]
   c. \[
   \begin{align*}
   0.5x + 1.5y &= 5 \\
   x + y &= -10
   \end{align*}
   \]

4. Classify each system as consistent or inconsistent. If a system is consistent, classify it as dependent or independent.
   a. \[
   \begin{align*}
   -3x + 2y &= 8 \\
   y &= 4 - x
   \end{align*}
   \]
   b. \[
   \begin{align*}
   6m + 3n &= 15 \\
   n &= -2m + 5
   \end{align*}
   \]
   c. \[
   \begin{align*}
   k &= 2j + 9 \\
   4j - 2k &= 3
   \end{align*}
   \]
Lesson 4.1 • Interpreting Graphs

Name ___________________________ Period ______________ Date ______________

1. Describe the pattern of the graph of each of the following situations as the graphs are read from left to right as increasing, decreasing, increasing and then decreasing, or decreasing and then increasing.
   a. The height of a child at birth and on each birthday from age 1 to age 6
   b. The height of a ball that is thrown upward from the top of a building from the time it is thrown until it hits the ground

2. For each of the situations described in Exercise 1, describe the real-world meaning of the vertical intercept of the graph.

3. Sketch a graph to match the description below.
   Increasing rapidly at a constant rate, then suddenly becoming constant, then decreasing rapidly at a constant rate

4. Sketch what you think is a reasonable graph for each relationship described. In each situation, identify the variables and label your axes appropriately.
   a. The temperature of a hot drink sitting on your desk
   b. Your speed as you cycle up a hill and down the other side
Lesson 4.2 • Function Notation

1. Determine whether or not each graph represents a function. Explain how you know.
   a. 
   b. 
   c. 

2. Find each of the indicated function values.
   a. If \( f(x) = -\sqrt{4x + 1} \), find \( f\left(-\frac{1}{4}\right) \), \( f(0) \), \( f(0.75) \), \( f(2) \), and \( f(12) \).
   b. If \( f(x) = \frac{2}{x-4} \), find \( f(-4) \), \( f(0) \), \( f(5) \), \( f(8) \), and \( f(24) \).

3. Use the graph at right to find each of the following.
   a. \( f(3) + f(-3) \) 
   b. \( f(f(10)) \)
   c. \( x \) when \( f(x) = -3 \) 
   d. \( x \) when \( f(x - 3) = 35 \)

4. Define variables and write a function that describes each situation.
   a. You drive on an interstate highway with your cruise control set at 65 miles per hour and do not need to stop or alter your speed.
   b. You rent a small moving van to move your belongings to your new apartment. The rental company charges $45 a day plus $0.22 a mile to rent the van.
Lesson 4.3 • Lines in Motion

1. Describe how each graph translates the graph of \( y = f(x) \).
   a. \( y = f(x) - 3 \)  
   b. \( y = f(x + 6) \)  
   c. \( y = 5 + f(x - 7) \)

2. Find each of the following.
   a. \( f(x - 2) \) if \( f(x) = -4x \)  
   b. \( 3 + f(x + 4) \) if \( f(x) = 2x \)  
   c. \( f(x - 5) \) if \( f(x) = 2x + 1 \)  
   d. \( 3 + f(x + 6) \) if \( f(x) = 8 - x \)

3. Write an equation for each line.
   a. The line \( y = -1.2x \) translated right 3 units

4. The graph of \( y = f(x) \) is shown at right. Write an equation for each related graph showing how the function has been translated.
   a. 
   b. 
   c. 
   d.
Lesson 4.4 • Translations and the Quadratic Family

1. Describe the translations of the graph of \( y = x^2 \) needed to produce the graph of each equation.
   a. \( y = x^2 - 6 \)  
   b. \( y = (x + 5)^2 \)  
   c. \( y = (x - 3)^2 - 9 \)

2. Find the vertex of each parabola.
   a. \( y = x^2 + 3 \)  
   b. \( y = (x - 2)^2 \)  
   c. \( y = -8 + (x + 5)^2 \)

3. Each parabola described is the graph of \( y = x^2 \). Write an equation for each parabola and sketch its graph.
   a. The parabola is translated horizontally \(-3\) units.
   b. The parabola is translated vertically \(1\) unit.
   c. The parabola is translated horizontally \(2\) units and vertically \(-3\) units.

4. Describe what happens to the graph of \( y = x^2 \) in the following situations.
   a. \( y \) is replaced with \((y + 1)\).  
   b. \( x \) is replaced with \((x - 5)\).

5. Solve.
   a. \( x^2 + 6 = 31 \)  
   b. \( x^2 - 12 = 52 \)  
   c. \((x - 3)^2 = 100 \)
   
   d. \((x + 7)^2 = 144 \)  
   e. \((x + 4)^2 - 5 = 31 \)  
   f. \(-20 + (x - 5)^2 = 3 \)  

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Lesson 4.5 • Reflections and the Square Root Family

1. Describe what happens to the graph of \( y = \sqrt{x} \) in each of the following situations.
   a. \( x \) is replaced with \((x + 6)\).
   b. \( y \) is replaced with \((y - 5)\).
   c. \( y \) is replaced with \((y + 1)\).
   d. \( x \) is replaced with \((x - 8)\).

2. Each graph below is a transformation of the graph of either the parent function \( y = x^2 \) or the parent function \( y = \sqrt{x} \). Write an equation for each graph.

3. Given the graph of \( y = f(x) \) at right, draw a graph of each of these related functions.
   a. \( y = -f(x) \)
   b. \( y = f(-x) \)
   c. \( y = -f(-x) \)
Lesson 4.6 • Dilations and the Absolute-Value Family

1. Each graph is a transformation of one of the parent functions you’ve studied. Write an equation for each graph.

2. Describe the transformations of the graph of \( y = |x| \) needed to produce the graph of each equation.
   a. \( y = |x - 3| \)
   b. \( \frac{y}{1.5} = \frac{|x|}{2} \)
   c. \( \frac{y + 4}{2} = |x - 1| \)

3. Find the vertex of the graph of each equation in Exercise 2 and sketch the graph.

4. Solve.
   a. \( |x - 5| - 7 = 0 \)
   b. \( 3|x - 5| - 2 = 10 \)
   c. \( \frac{x}{2} + 5 = 12 \)

5. Solve each equation for \( y \).
   a. \( \frac{y}{2} = \frac{|x|}{4} \)
   b. \( \frac{y - 3}{2} = (x + 1)^2 \)
   c. \( \frac{y + 1}{-3} = \sqrt{x + 2} \)
Lesson 4.7 • Transformations and the Circle Family

1. Write an equation for each circle.

2. If \( f(x) = \sqrt{1 - x^2} \), write an equation for each of the following related functions.
   a. \( -f(x) \)   b. \( f(-x) \)   c. \( 2f(x) \)   d. \( f(2x) \)

3. Without graphing, find the \( x \)- and \( y \)-intercepts of the graph of each equation.
   a. \( y = -\sqrt{1 - x^2} \)
   b. \( y = -2\sqrt{1 - x^2} \)
   c. \( y = \sqrt{1 - (2x)^2} \)
   d. \( y = -2\sqrt{1 - (4x)^2} \)
   e. \( y = -\sqrt{1 - \left(\frac{x}{3}\right)^2} \)
   f. \( y = 2\sqrt{1 - \left(\frac{x}{4}\right)^2} \)

4. Write an equation for each transformation of the unit circle, and identify its graph as a circle or an ellipse. Then sketch the graph.
   a. Replace \( x \) with \( \frac{x}{2} \) and \( y \) with \( \frac{y}{2} \).
   b. Replace \( x \) with \( \frac{x}{4} \) and \( y \) with \( \frac{y}{3} \).
Lesson 4.8 • Compositions of Functions

1. The functions $f$ and $g$ are defined by sets of input and output values.
   
   $f = \{(5, 0), (-1, 1), (-3, 4), (1, 2), (3, 4), (-2, 6)\}$
   
   $g = \{(4, -1), (0, -2), (1, -1), (2, -2), (6, 0)\}$
   
   a. What is the domain of $f$?  
   b. What is the range of $g$?  
   c. Find $f(g(4))$.
   
   d. Find $g(f(-3))$.  
   e. Find $f(g(f(5)))$.  
   f. Find $g(f(g(0)))$.

2. Use these three functions to find each value: $f(x) = -3x + 5$, $g(x) = (x - 2)^2$, $h(x) = x^2 + 4$.
   
   a. $g(2x) + 1$  
   b. $h(f(7))$  
   c. $h(g(f(0)))$  
   d. $g(h(a))$

3. Marla, Shamim, and Julie went out for dinner together. The sales tax on the meal was 6%, and they agreed to leave a 15% tip. Marla thought they should calculate the tip by finding 15% of the total bill, including the sales tax. Shamim thought they should calculate the tip by finding 15% of the bill before the tax was added. Julie thought it wouldn’t make any difference. Let $x$ represent the cost of the meal in dollars before tax and tip are added.
   
   a. Find a function $f$ that gives the cost of the meal, including sales tax but not the tip.
   
   b. Find a function $g$ that gives the amount of the tip calculated the way Shamim suggested.
   
   c. Use composition to find a function that gives the amount of the tip calculated the way Marla suggested.
   
   d. If the cost of the meal before tax was $50, find the amount they will leave as a tip, calculated Marla’s way and Shamim’s way.
Lesson 5.1 • Exponential Functions

1. Evaluate each function at the given value. Round to four decimal places if necessary.
   a. \( r(t) = 325(1 + 0.035)^t \), \( t = 8 \)
   b. \( j(x) = 59.5(1 - 0.095)^x \), \( x = 10 \)

2. Record the next three terms for each sequence. Then write an explicit function for the sequence.
   a. \( a_0 = 12 \)
   \( a_n = 0.8a_{n-1} \) where \( n \geq 1 \)
   b. \( u_0 = 50.5 \)
   \( u_n = 2.1u_{n-1} \) where \( n \geq 1 \)

3. Evaluate each function at \( x = 0 \), \( x = 1 \), and \( x = 2 \) and write a recursive formula for the pattern. Then, indicate whether each equation is a model for exponential growth or decay.
   a. \( f(x) = 2000(0.9)^x \)
   b. \( f(x) = 3000(1 + 0.001)^x \)
   c. \( f(x) = 0.1(1 - 0.5)^x \)

4. Calculate the ratio of the second term to the first term, and express the answer as a decimal value. State the percent increase or decrease.
   a. 80, 60
   b. 36, 32
   c. 63, 100.8

5. Rohit bought a new car for $17,500. The value of the car is depreciating at a rate of 16% a year.
   a. Write a recursive formula that models this situation. Let \( u_0 \) represent the purchase price, \( u_i \) represent the value of the car after 1 year, and so on.
   b. Define variables and write an exponential equation that models this situation.
1. Rewrite each expression as a fraction without exponents or as an integer. Using your calculator, verify that your answer is equivalent to the original expression.
   a. \(3^{-2}\)
   b. \(7^{-3}\)
   c. \(-4^{-4}\)
   d. \((-5)^{-3}\)
   e. \(-\left(\frac{3}{5}\right)^{-2}\)
   f. \(-\left(\frac{5}{6}\right)^{-2}\)

2. Rewrite each expression in the form \(x^n\) or \(ax^n\).
   a. \(4x^0 \cdot 9x^8\)
   b. \((8x^{-6})(-15x^{-14})\)
   c. \(\frac{x^9}{x^{-9}}\)
   d. \(\frac{-88x^{10}}{-8x^3}\)
   e. \(\left(-\frac{35x^7}{-7x^2}\right)^3\)
   f. \(\left(-\frac{40x^{-8}}{-8x^{-2}}\right)^{-3}\)

3. Solve.
   a. \(2^x = \frac{1}{32}\)
   b. \(125^x = 25\)
   c. \(\left(\frac{4}{9}\right)^x = \frac{81}{16}\)

4. Solve each equation for positive values of \(x\). If answers are not exact, approximate to two decimal places.
   a. \(6x^{1.5} = 80\)
   b. \(20x^{1/2} - 8 = 4.5\)
   c. \(5x^{-1/3} = 0.06\)
   d. \(8x^9 = 6x^6\)
   e. \(15x^{-3} = 10x^{-2}\)
   f. \(200x^{-1} = 125x^{-3}\)
Lesson 5.3 • Rational Exponents and Roots

1. Identify each function as a power function, an exponential function, or neither of these. (The function may be translated, stretched, or reflected.)
   a. $f(x) = 0.5x^3 - 4$
   b. $f(x) = \frac{1}{3^x}$
   c. $f(x) = \frac{1}{x} + 2$

2. Rewrite each expression in the form $b^x$ in which $x$ is a rational exponent.
   a. $\sqrt[3]{c^3}$
   b. $(\sqrt[4]{d})^4$
   c. $\frac{1}{\sqrt[5]{r^5}}$

3. Solve each equation for positive values of $x$. If answers are not exact, approximate to the nearest hundredth.
   a. $\sqrt[5]{x^3} = 27$
   b. $\frac{1}{\sqrt{x}} = 0.77$
   c. $4\sqrt[3]{x} + 18 = 32$
   d. $\sqrt[5]{x^3} - 23 = -15$
   e. $\sqrt[3]{4x^2} + 8.5 = 19.8$
   f. $\sqrt[5]{x^5} = 12.75$

4. Each of the following graphs is a transformation of the power function $y = x^{3/2}$. Write the equation for each curve.
Lesson 5.4 • Applications of Exponential and Power Equations

1. Solve each equation for positive values of \( x \). If answers are not exact, approximate to the nearest hundredth.
   a. \( \sqrt[3]{x} = 2.6 \)  
   b. \( x^{-1/4} = 0.2 \)  
   c. \( 0.75x^5 - 8 = -3 \)
   
   d. \( 4(x^{5/6} + 7) = 159 \)  
   e. \( 224 = 200 \left( 1 + \frac{x}{4} \right)^9 \)  
   f. \( 1500 \left( 1 + \frac{x}{12} \right)^{6.5} = 1525 \)

2. Rewrite each expression in the form \( ax^n \).
   a. \( (8x^9)^{2/3} \)  
   b. \( (81x^{12})^{3/4} \)  
   c. \( (49x^{-10})^{1/2} \)
   
   d. \( (-27x^{-9})^{4/3} \)  
   e. \( (100,000x^{10})^{3/5} \)  
   f. \( (-125x^{-15})^{1/3} \)

3. Give the average annual rate of inflation for each situation described. Give your answers to the nearest tenth of a percent.
   a. The cost of a movie ticket increased from $6.00 to $8.50 over 10 years.
   b. The monthly rent for Hector’s apartment increased from $650 to $757 over 4 years.

4. The population of a small town has been declining because jobs have been leaving the area. The population was 23,000 in 2002 and 18,750 in 2007. Assume that the population is decreasing exponentially.
   a. Define variables and write an equation that models the population in this town in a particular year.
   b. Use your model to predict the population in 2010.
   c. According to your model, in what year will the population first fall below 12,000?
Lesson 5.5 • Building Inverses of Functions

1. Each of the functions below has an inverse that is also a function. Find four points on the graph of each function \( f \), using the given values of \( x \). Use these points to find four points on the graph of \( f^{-1} \).
   a. \( f(x) = 3x - 4; x = -2, 0, \frac{4}{3}, 4 \)
   b. \( f(x) = x^3 - 2; x = -3, -1, 2, 5 \)

2. For each function below, determine whether or not the inverse of the function is a function. Find the equation of the inverse and graph both equations on the same axes.
   a. \( y = -2x + 5 \)
   b. \( y = |x| \)
   c. \( y = x^2 - 4 \)

3. Balloons and Laughs Inc. is a small company that entertains at children’s birthday parties. B & L uses a complicated formula to calculate its prices, taking into account all of its costs. The price equation is \( p(x) = 4\sqrt[3]{(8x + 3)^2 + 25} \), where \( x \) is the number of person-hours supplied for the party at a price of \( p(x) \). For example, if \( x = 4 \), four clowns will come for one hour, two clowns will come for two hours, or one clown will come for four hours.
   a. What is the price if two clowns come to a party for 90 minutes?
   b. Many customers want to know what they can get for a particular amount of money. Rewrite the price equation for B & L so that the company can input the amount of money a customer wants to spend and the output will be the number of person-hours he or she will get for the money. Call the new function \( p^{-1}(x) \).
   c. B & L’s Ultimate Party costs $125. How many person-hours do you get at an Ultimate Party?
Lesson 5.6 • Logarithmic Functions

1. Rewrite each logarithmic equation in exponential form using the definition of logarithm. Then solve for x.
   a. \( \log_3 \frac{1}{81} = x \)
   b. \( \log_x \sqrt[4]{12} = \frac{1}{4} \)
   c. \( x = \log_4 32 \)
   d. \( \log x = 1 \)
   e. \( 3 = \log_4 125 \)
   f. \( \log_{20} x = 1 \)

2. Find the exact value of each logarithm without using a calculator. Write answers as integers or fractions in lowest terms.
   a. \( \log_3 81 \)
   b. \( \log_5 \sqrt{5} \)
   c. \( \log_3 \frac{1}{3} \)
   d. \( \log_{20} \frac{1}{32} \)
   e. \( \log_8 4 \)
   f. \( \log_{1,000,000,000} 1 \)

3. Each graph is a transformation of either \( y = 10^x \) or \( y = \log x \). Write the equation for each graph.
   a. 
   ![Graph A]
   b. 
   ![Graph B]
   c. 
   ![Graph C]

4. Use the change-of-base property to solve each equation. (Round to four decimal places.)
   a. \( \log_3 120 = x \)
   b. \( \log_3 0.9 = x \)
   c. \( 4^x = 99 \)
   d. \( 6^x = 729 \)
   e. \( 7^x = 4.88 \)
   f. \( 12^x = 5.75 \)
Lesson 5.7 • Properties of Logarithms

1. Use the properties of logarithms to rewrite each expression as a single logarithm.
   a. \( \log 21 - \log 7 \)
   b. \(-4 \cdot \log 2\)
   c. \(-2 \cdot \log 5 + 4 \cdot \log 5\)

2. Write each expression as a sum or difference of logarithms (or constants times logarithms). Simplify the result if possible.
   a. \( \log_5 \frac{a \sqrt{b}}{c^4} \)
   b. \( \log_4 \left( \sqrt{r} \cdot \sqrt[3]{s} \cdot \sqrt[4]{t^3} \right) \)
   c. \( \log_3 \left( \sqrt[3]{abc} \right) \)

3. Determine whether each equation is true or false.
   a. \( \log 8 = \frac{\log 32}{\log 4} \)
   b. \( \log_5 9 - \log_5 2 = \log_5 4.5 \)
   c. \( \log \frac{1}{5} = \frac{1}{\log 5} \)
   d. \( \log 4 = \frac{2}{3} \log 8 \)
   e. \( \log \sqrt[3]{5} = \frac{1}{3} \log 5 \)
   f. \( \log_2 \frac{1}{81} = -2 \log_2 9 \)
   g. \( \log \sqrt{6} = -2 \log 6 \)
   h. \( \log_3 15 - \log_3 5 = 1 \)

4. Change the form of each expression below using definitions or properties of logarithms or exponents. Name each definition or property you use.
   a. \( \log r - \log s \)
   b. \( \frac{1}{a^b} \)
   c. \( a^{a+b} \)
   d. \( \log_b x^m \)
   e. \( (cd)^m \)
   f. \( \log_b xy \)
   g. \( \left( \frac{r}{s} \right)^m \)
   h. \( c^{m/n} \)
   i. \( \frac{\log_a x}{\log_a y} \)
Lesson 5.8 • Applications of Logarithms

1. Solve each equation. Round answers that are not exact to four decimal places.
   a. \(19,683 = 3^x\)  
   b. \(9.5(8^x) = 220\)  
   c. \(0.405 = 15.6(0.72)^x\)

2. Suppose that you invest $5,000 in a savings account. How long would it take you to double your money under each of the following conditions?
   a. 5% interest compounded annually
   b. 3.6% interest compounded monthly

3. The Richter scale rating of the magnitude of an earthquake is given by the formula \(\log \left( \frac{I}{I_0} \right)\), where \(I_0\) is a certain small magnitude used as a reference point. (Richter scale ratings are given to the nearest tenth.)
   a. A devastating earthquake, which measured 7.4 on the Richter scale, occurred in western Turkey in 1999. Express the magnitude of this earthquake as a multiple of \(I_0\).
   b. Another earthquake occurred in 1998, centered in Adana, Turkey. This earthquake measured 6.3 on the Richter scale. Express the magnitude of this earthquake as a multiple of \(I_0\).
   c. Compare the magnitudes of the two earthquakes.

4. The population of an animal species introduced into an area sometimes increases rapidly at first and then more slowly over time. A logarithmic function models this kind of growth. Suppose that a population of \(N\) deer in an area \(t\) months after the deer are introduced is given by the equation \(N = 325 \log(4t + 2)\). How long will it take for the deer population to reach 800? Round to the nearest whole month.
Lesson 6.1 • Matrix Representations

1. A survey of registered voters showed that of those people who voted in the 2008 presidential election, 87% expected to vote in the 2012 election, whereas of those who did not vote in the 2008 presidential election, 22% expected to vote in the 2012 election.
   a. Complete the following transition diagram to represent the survey results.

   ![Transition Diagram]

   b. Write a transition matrix that conveys the same information as your transition diagram. List voters first and nonvoters second.

   c. Suppose that the survey included 300 people who voted in 2008 and 300 who did not. Based on this information, how many of these people would you expect to vote in 2012?

2. Matrix \([M]\) represents the vertices of \(\triangle DEF\).

   \[
   [M] = \begin{bmatrix}
   -5 & 2 & 4 \\
   3 & 0 & -6
   \end{bmatrix}
   \]

   a. Name the coordinates of the vertices and graph the triangle.

   b. What matrix represents the image of \(\triangle DEF\) after a translation left 3 units?

   c. What matrix represents the image of \(\triangle DEF\) after a translation right 4 units and down 2 units?
Lesson 6.2 • Matrix Operations

Name ________________________________  Period _________  Date ______________

1. Find the missing values.
   a. \[
      [3 \quad 12 \quad -8] + [9 \quad -12 \quad 13] = [x \quad y \quad z]
   \]
   b. \[
      -5 \begin{bmatrix}
      3.8 & -5.2 \\
      -1.9 & 0.8
      \end{bmatrix} = \begin{bmatrix}
      n_{11} & n_{12} \\
      n_{21} & n_{22}
      \end{bmatrix}
   \]

2. Perform matrix arithmetic in 2a–d. If a particular operation is impossible, explain why.
   a. \[
      \begin{bmatrix}
      8.5 & 4.2 \\
      3.6 & -2.7
      \end{bmatrix} - \begin{bmatrix}
      7.9 & 8.8 \\
      2.9 & -0.9
      \end{bmatrix}
   \]
   b. \[
      \begin{bmatrix}
      1 & 0 & 2 \\
      -1 & 4 & 5 \\
      0 & 1 & 3
      \end{bmatrix} \begin{bmatrix}
      10 \\
      -8 \\
      4
      \end{bmatrix}
   \]
   c. \[
      \begin{bmatrix}
      2 & -5 \\
      3 & 5 \\
      -1 & 4
      \end{bmatrix}
   \]
   d. \[
      0.5 \begin{bmatrix}
      20 & -10 \\
      16 & 14
      \end{bmatrix} + 2.5 \begin{bmatrix}
      12 & -8 \\
      -16 & 30
      \end{bmatrix}
   \]

3. This matrix represents a triangle.
   \[
   \begin{bmatrix}
   -2 & 2 & -1 \\
   -3 & 0 & 3
   \end{bmatrix}
   \]
   a. Graph the triangle. Label the vertices with their coordinates.
   b. Find the result of this matrix multiplication:
      \[
      \begin{bmatrix}
      0 & 1 \\
      1 & 0
      \end{bmatrix} \begin{bmatrix}
      -2 & 2 & -1 \\
      -3 & 0 & 3
      \end{bmatrix}
   \]
   c. Graph the image represented by the matrix in 3b.
   d. Describe the transformation.
Lesson 6.3 • Solving Systems with Inverse Matrices

1. Rewrite each matrix equation as a system of equations.
   a. \[
   \begin{bmatrix}
   4 & -3 \\
   2 & 6
   \end{bmatrix}
   \begin{bmatrix}
   x \\
   y
   \end{bmatrix}
   =
   \begin{bmatrix}
   5 \\
   -1
   \end{bmatrix}
   \]
   b. \[
   \begin{bmatrix}
   0.5 & 0.8 \\
   0.1 & -0.2
   \end{bmatrix}
   \begin{bmatrix}
   x \\
   y
   \end{bmatrix}
   =
   \begin{bmatrix}
   4 \\
   1.5
   \end{bmatrix}
   \]

2. Rewrite the system of equations in matrix form.
   \[
   \begin{align*}
   2x + 3y - z &= -5 \\
   3x + 4z &= -1 \\
   y - 2z &= -3
   \end{align*}
   \]

3. Determine whether or not the matrices in each pair are inverses.
   a. \[
   \begin{bmatrix}
   2 & -3 \\
   5 & -8
   \end{bmatrix},
   \begin{bmatrix}
   -8 & 3 \\
   -5 & 2
   \end{bmatrix}
   \]
   b. \[
   \begin{bmatrix}
   1 & 0 & -1 \\
   0 & 1 & 1 \\
   1 & 1 & 1
   \end{bmatrix},
   \begin{bmatrix}
   0 & -1 & 1 \\
   1 & 2 & -1 \\
   -1 & -1 & 1
   \end{bmatrix}
   \]

4. Find the inverse of each matrix if the inverse exists.
   a. \[
   \begin{bmatrix}
   3 & -5 \\
   -2 & 4
   \end{bmatrix}
   \]
   b. \[
   \begin{bmatrix}
   4 & -6 \\
   2 & -3
   \end{bmatrix}
   \]
   c. \[
   \begin{bmatrix}
   1 & 0 & -1 \\
   2 & 1 & 0 \\
   0 & -2 & 1
   \end{bmatrix}
   \]

5. Rolando bought 4 CDs and 5 DVDs for a total of $146. Suzanna bought 6 CDs and 3 DVDs for a total of $138. All the CDs were one price and all the DVDs were one price.
   a. Let \( c \) represent the price of a CD and \( d \) represent the price of a DVD. Write a system of equations that you can use to find the prices. Then, rewrite your system in matrix form, \([A][X] = [B]\).
   b. Write an equivalent matrix equation in the form \([X] = [A]^{-1}[B]\).
   c. Use matrix multiplication to find \([X]\). What was the cost of one CD and the cost of one DVD?
Lesson 6.4 • Row Reduction Method

Name ____________________________ Period ________ Date ____________

1. Write a system of equations for each augmented matrix.
   a. \[
   \begin{bmatrix}
   3 & -1 & 5 \\
   1 & 4 & -2
   \end{bmatrix}
   \]
   b. \[
   \begin{bmatrix}
   4 & -1 & 1 & 6 \\
   -2 & 3 & -2 & 5 \\
   3 & 4 & -4 & 0
   \end{bmatrix}
   \]

2. Write an augmented matrix for each system.
   a. \[
   \begin{align*}
   x + y - z &= 0 \\
   2x + 3y - 3z &= -3 \\
   -x - 2y + 2z &= 3
   \end{align*}
   \]
   b. \[
   \begin{align*}
   2x + z &= 8 \\
   3y - 4z &= -1 \\
   4x - y &= 3
   \end{align*}
   \]

3. Perform the given row operation on each matrix.
   a. \[
   \begin{bmatrix}
   4 & -2 & 5 \\
   0 & -3 & 12
   \end{bmatrix}; \; \frac{R_2}{3} \rightarrow R_2
   \]
   b. \[
   \begin{bmatrix}
   1 & 3 & 5 & -2 \\
   3 & -4 & 2 & 0 \\
   -2 & 4 & 6 & 1
   \end{bmatrix}; \; -3R_1 + R_2 \rightarrow R_2
   \]

4. Three kinds of tickets were sold for a concert. Main floor tickets cost $35, balcony tickets cost $25, and gallery tickets cost $15. The box office sold 475 tickets for a total of $13,275. There were 45 more main floor tickets sold than balcony tickets.
   a. Let \(m\) represent the number of main floor tickets sold, \(b\) represent the number of balcony tickets sold, and \(g\) represent the number of gallery tickets sold. Write a system of equations that you can use to find how many tickets of each kind were sold. Then, rewrite your system of equations as an augmented matrix.
   
   b. Apply row operations to your matrix to transform it into reduced row-echelon form. How many tickets of each kind were sold?
Lesson 6.5 • Systems of Inequalities

Name _____________________________ Period ___________ Date ______________

1. Solve each inequality for \( y \).
   a. \( 4x - 5y \leq 20 \)  
   b. \( 3 + 2(x - 4y) > 12 \)

2. Graph each linear inequality on the coordinate plane.
   a. \( y > \frac{1}{3}x - 3 \)  
   b. \( 3x - 5y \geq 0 \)  
   c. \( 4y - 2x \leq -8 \)

3. Graph the feasible region of each system of inequalities. Find the coordinates of each vertex.
   a. \[
   \begin{align*}
   x - 3y &\geq -11 \\
   x - 3y &\leq -2 \\
   1 &\leq x \leq 4
   \end{align*}
   \]
   b. \[
   \begin{align*}
   y &\leq -|x| + 4 \\
   y &\leq 3x \\
   x &\geq 0 \\
   y &\geq 0
   \end{align*}
   \]
   c. \[
   \begin{align*}
   y &\leq \sqrt{25 - x^2} \\
   y &\geq 0.75x \\
   x &\geq 0
   \end{align*}
   \]

4. Leo is taking an algebra test containing computation problems worth 5 points each and application problems worth 8 points each. Leo needs to score at least 83 points on the test to maintain his B average. Let \( c \) represent the number of computation problems he answers correctly and \( a \) represent the number of application problems he answers correctly. Write an inequality to represent the constraint.

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Lesson 6.6 • Linear Programming

1. A yoga teacher sells yoga mats for $25 each and yoga DVDs for $14 each. If \( x \) represents the number of mats she sells and \( y \) represents the number of DVDs she sells, write an expression that represents the amount of money she receives from selling mats and DVDs. Tell whether the expression should be minimized or maximized.

2. You would like to maximize profits at your bakery, which makes decorated sheet cakes for parties in two sizes, a full sheet and a half sheet. A batch of 12 full-sheet cakes takes 3.5 hours of oven time and 3 hours of decorating time, whereas a batch of 20 half-sheet cakes takes 5 hours of oven time and 4 hours of decorating time. The oven is available for a maximum of 21 hours a day, and the decorating room is available for 14 hours a day. Let \( x \) represent the number of batches of sheet cakes that the bakery produces in one day, and let \( y \) represent the number of batches of half-sheet cakes. The bakery makes a profit of $30 on each batch of full-sheet cakes and $35 on each batch of half-sheet cakes.

   a. Write a constraint about oven time.

   b. Write a constraint about decorating time.

   c. Write a system of inequalities that includes the constraints you have found and any commonsense constraints.

   d. Graph the feasible region and find the vertices.

   e. Find the profit at each vertex.

   f. How many batches of each size of cake should the bakery make in one day to maximize profit? What is the maximum profit?
Lesson 7.1 • Polynomial Degree and Finite Differences

1. Identify the degree of each polynomial.
   a. $x^5 - 1$  
   b. $0.2x - 1.5x^2 + 3.2x^3$  
   c. $250 - 16x^2 + 20x$

2. Determine which of the expressions are polynomials. For each polynomial, state its degree and write it in general form. If it is not a polynomial, explain why not.
   a. $0.2x^3 + 0.5x^4 + 0.6x^2$  
   b. $x - \frac{1}{x^2}$  
   c. 25

3. The figures below show why the numbers in the sequence 1, 3, 6, 10, . . . are called *triangular numbers*.

   ![Triangular Numbers Diagram]

   a. Complete the table.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$th triangular number</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Calculate the finite differences for the completed table.

   c. What is the degree of the polynomial function that you would use to model this data set?

   d. Write a polynomial function $t$ that gives the $n$th triangular number as a function of $n$. (*Hint: Create and solve a system of equations to find the coefficients.*)
Lesson 7.2 • Equivalent Quadratic Forms

1. Identify each quadratic function as being in general form, vertex form, factored form, or none of these forms.
   a. \( y = 3x^2 - 4x + 5 \)  
   b. \( y = (x - 2.5)^2 + 7.5 \)  
   c. \( y = -1.5x(x - 2) \)

2. Convert each quadratic function to general form.
   a. \( y = (x - 3)^2 \)  
   b. \( y = -5(x + 3)(x - 2) - 30 \)  
   c. \( y = 3(x - 1.5)^2 - 10 \)

3. Find the vertex of the graph of each quadratic function.
   a. \( y = -x^2 \)  
   b. \( y = -(x - 1)^2 + 6 \)  
   c. \( y = 6.5 + 0.5(x + 4)^2 \)

4. Find the zeros of each quadratic function.
   a. \( y = -2(x - 1)(x + 6) \)  
   b. \( y = 0.5x(x - 5) \)  
   c. \( y = (x - 7.5)^2 \)

5. Consider this table of values generated by a quadratic function.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( -3 )</th>
<th>( -2.5 )</th>
<th>( -2 )</th>
<th>( -1.5 )</th>
<th>( -1 )</th>
<th>( -0.5 )</th>
<th>( 0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>( -0.5 )</td>
<td>( -3 )</td>
<td>( -4.5 )</td>
<td>( -5 )</td>
<td>( -4.5 )</td>
<td>( -3 )</td>
<td>( -0.5 )</td>
</tr>
</tbody>
</table>

a. What is the line of symmetry for the graph of the quadratic function?

b. Identify the vertex of the graph of this quadratic function, and determine whether it is a maximum or a minimum.

c. Use the table of values to write the quadratic function in vertex form.
Lesson 7.3 • Completing the Square

1. Factor each quadratic expression.
   a. \(x^2 + 10x + 25\)  
   b. \(x^2 - x + \frac{1}{4}\)  
   c. \(9x^2 - 24xy + 16y^2\)

2. What value is required to complete the square?
   a. \(x^2 - 18x + \underline{\text{____}}\)  
   b. \(x^2 - 5x + \underline{\text{____}}\)  
   c. \(x^2 + 4.3x + \underline{\text{____}}\)

3. Convert each quadratic function to vertex form by completing the square.
   a. \(y = x^2 + 14x + 50\)  
   b. \(y = 5x^2 - 10x - 3\)  
   c. \(y = 2x^2 + 5x\)

4. Find the vertex of the graph of each quadratic function, and state whether the vertex is a maximum or a minimum.
   a. \(y = (x - 2)(x + 6)\)  
   b. \(y = -3.5x^2 - 7x\)  
   c. \(y = x^2 + 9x - 10\)

5. Rewrite each expression in the form \(ax^2 + bx + c\), and then identify the coefficients \(a\), \(b\), and \(c\).
   a. \(-6 + 3x^2 + 6x + 8\)  
   b. \(-2x(x - 8)\)  
   c. \((2x - 3)(x + 5)\)

6. A ball is thrown up and off the roof of a 75 m tall building with an initial velocity of 14.7 m/s.
   a. Let \(t\) represent the time in seconds and \(h\) represent the height of the ball in meters. Write an equation that models the height of the ball.

   b. At what time does the ball reach maximum height? What is the ball’s maximum height?

   c. At what time does the ball hit the ground?
Lesson 7.4 • The Quadratic Formula

1. Evaluate each expression. Round your answers to the nearest thousandth.
   a. \[\frac{-6 + \sqrt{6^2 - 4(1)(-5)}}{2(1)}\]
   b. \[\frac{4 - \sqrt{(-4)^2 - 4(2)(1)}}{2(2)}\]
   c. \[\frac{5 + \sqrt{(-5)^2 - 4(4)(-3)}}{2(4)}\]
   d. \[\frac{-10 - \sqrt{10^2 - 4(2)(5)}}{2(2)}\]

2. Solve by any method. Give your answers in exact form.
   a. \[x^2 + 3x - 10 = 0\]
   b. \[2x^2 - 5x = 12\]
   c. \[25x^2 - 49 = 0\]
   d. \[4x^2 + 7x - 1 = 0\]
   e. \[x^2 = 5.8x\]
   f. \[x^2 - 48 = 0\]

3. Use the quadratic formula to find the zeros of each function. Then, write each equation in factored form, \(y = a(x - r_1)(x - r_2)\), where \(r_1\) and \(r_2\) are the zeros of the function.
   a. \(y = x^2 + 5x - 24\)
   b. \(y = 2x^2 - 8x + 6\)
   c. \(y = 4x^2 + 2x - 2\)

4. Write a quadratic function in general form that satisfies the given conditions.
   a. \(a = -1; x\)-intercepts of graph are \(-4\) and \(-2\)
   b. \(x\)-intercepts of graph are 0 and 13; graph contains point (2, 22)
   c. \(x\)-intercept of graph is 4.8; \(y\)-intercept is \(-5.76\)
Lesson 7.5 • Complex Numbers

1. Add, subtract, or multiply.
   a. \((-5 + 6i) - (1 - i)\)       b. \((-2.4 - 5.6i) + (5.9 + 1.8i)\)
   c. \(-4i(-6 + i)\)            d. \((2.5 + 1.5i)(3.4 - 0.6i)\)

2. Find the conjugate of each complex number.
   a. \(5 - 4i\)                  b. \(7i\)                      c. \(-3.25 + 4.82i\)

3. Rewrite each quotient in the form \(a + bi\).
   a. \(\frac{2}{3 + i}\)           b. \(\frac{1 + i}{1 - i}\)      c. \(\frac{3 + 5i}{6i}\)

4. Solve each equation. Use substitution to check your solutions. Label each solution as real, imaginary, and/or complex.
   a. \(x^2 - 2x + 5 = 0\)         b. \(x^2 + 7 = 0\)             c. \(x(x - 5) = 1\)
   d. \(x^2 + x + 1 = 0\)          e. \(4x^2 + 9 = 0\)          f. \((x + 7)(x - 3) = 5 - 2x\)

5. Write a quadratic function in general form that has the given zeros and leading coefficient of 1.
   a. \(x = -4, x = 7\)         b. \(x = 11i, x = -11i\)
   c. \(x = -2 + 3i, x = -2 - 3i\)

6. Name the complex number associated with each point, A through C, on the complex plane shown.
Lesson 7.6 • Factoring Polynomials

1. Without graphing, find the x-intercepts and the y-intercept for the graph of each equation.
   a. \( y = -(x - 8)^2 \)  
   b. \( y = 3(x + 4)(x + 2) \)  
   c. \( y = 0.75x(x - 2)(x + 6) \)

2. Write the factored form of the quadratic function for each graph. Don’t forget the vertical scale factor.
   a. 
   b. 

3. Convert each polynomial function to general form.
   a. \( y = -2(x - 2.5)(x + 2.5) \)  
   b. \( y = -0.5(x + 3)^2 \)  
   c. \( y = -x(x + 12)(x - 12) \)

4. Write each polynomial as a product of factors. Some factors may include irrational numbers.
   a. \( x^2 - 14x + 49 \)  
   b. \( x^3 - 3x^2 + 2x \)  
   c. \( x^2 + 169 \)
   d. \( x^2 - 15 \)  
   e. \( x^4 - 10x^2 + 9 \)  
   f. \( 3x^3 + 3x^2 - 30x + 24 \)

5. Sketch a graph for each situation if possible.
   a. A quadratic function with two real zeros, whose graph has the line \( x = 2 \) as its axis of symmetry
   b. A cubic function with three real zeros, whose graph has a positive y-intercept
1. Refer to these two graphs of polynomial functions.

i. 

\[ \begin{array}{c}
\text{Graph 1} \\
(3, 0) \quad (2, 0)
\end{array} \]

\[ \begin{array}{c}
\text{Graph 2} \\
(0, 4)
\end{array} \]

a. Identify the zeros of each function.

b. Find the \( y \)-intercept of each graph.

c. Identify the lowest possible degree of each polynomial function.

d. Write the factored form for each polynomial function. Check your work by graphing on your calculator.

2. Write a polynomial function with the given features.

a. A quadratic function whose graph has vertex \((3, -8)\), which is a minimum, and two \( x \)-intercepts, one of which is 5

b. A fourth-degree polynomial function with two double roots, 0 and 2, and whose graph contains the point \((1, -1)\)

3. Write the lowest-degree polynomial function that has the given set of zeros and whose graph has the given \( y \)-intercept. Write each polynomial function in factored form. Give the degree of each function.

a. Zeros: \( x = -3, x = 5 \); \( y \)-intercept: -30

b. Zeros: \( x = \pm 2i, x = -2 \) (double root), \( x = 5 \); \( y \)-intercept: 80
Lesson 7.8 • More About Finding Solutions

1. Divide.
   a. $x - 2 | 3x^3 - 8x^2 - 11x + 30$
   b. $x - 4 | x^4 - 13x^2 - 48$

2. Varsha started out dividing two polynomials by synthetic division this way:
   
   \[ -3 \div 3 \quad -5 \quad 0 \quad -35 \quad 7 \]
   
   a. Identify the dividend and divisor.
   
   b. Write the numbers that will appear in the second line of the synthetic division.
   
   c. Write the numbers that will appear in the last line of the synthetic division.
   
   d. Write the quotient and remainder for this division.

3. In each division problem, use the polynomial that defines $P$ as the dividend and the binomial that defines $D$ as the divisor. Write the result of the division in the form $P(x) = D(x) \cdot Q(x) + R$, where the polynomial that defines $Q$ is the quotient and $R$ is an integer remainder. (It is not necessary to write the remainder if $R = 0$.)
   a. $P(x) = 2x^2 - 9x + 2; D(x) = x - 5$
   b. $P(x) = 2x^3 - 5x^2 + 8x - 5; D(x) = x - 1$

4. Make a list of the possible rational roots of each equation.
   a. $x^3 + x^2 - 10x + 8 = 0$
   b. $2x^3 - 3x^2 - 17x + 30 = 0$

5. Find all the zeros of each polynomial function. Then write the function in factored form.
   a. $y = x^3 - 5x^2 + 9x - 45$
   b. $y = 6x^3 + 17x^2 + 6x - 8$
Lesson 8.1 • Using the Distance Formula

1. Find the exact distance between each pair of points.
   a. (0, 0) and (5, 12)  
   b. (2, 8) and (6, 11)  
   c. (−2, 5) and (2, 7)  
   d. (4, −7) and (8, −15)  
   e. (3a, 8) and (−2a, 5)  
   f. (1/2, 1/4) and (−1/2, 9/4)  

2. Make a sketch of the situation and find the possible values of x or y.
   a. The distance between the points (−4, 6) and (2, y) is 10 units.
      
   b. The distance between the points (4, −5) and (x, 3) is 11 units.
      
3. Make a sketch of the situation and find an equation of the locus of points that satisfies the given condition.
   a. The points that are 5 units from (−2, 3)
      
   b. The points that are equidistant from (0, 0) and (2, 5)
      
   c. The points that are twice as far from (−9, 0) as they are from (0, 0)
Lesson 8.2 • Circles and Ellipses

1. Find the center and radius of each circle.
   a. $x^2 + y^2 = 16$
   b. $(x - 3)^2 + y^2 = 100$
   c. $(x - 0.5)^2 + (y + 0.5)^2 = 0.25$
   d. $(x + \frac{1}{3})^2 + (y - \frac{2}{3})^2 = \frac{25}{49}$

2. Find the center, horizontal scale factor, and vertical scale factor for each ellipse.
   a. $\left(\frac{x}{3}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$
   b. $\left(\frac{x - 2}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$
   c. $\frac{(x + 5)^2}{9} + \frac{(y - 4)^2}{25} = 1$

3. Sketch each ellipse in Exercise 2. Give the exact coordinates of the endpoints of the major and minor axes, and the foci.

4. Write an equation in standard form for each graph.
   a. 
   b. 
   c. 

$\frac{x^2}{36} + \frac{y^2}{25} = 1$
Lesson 8.3 • Parabolas

1. For each parabola described, use the information given to find the location of the missing feature. It may help to draw a sketch.
   a. If the vertex is (0, 0) and the focus is (4, 0), where is the directrix?
   b. If the vertex is (5, 0) and the directrix is \( x = 1.5 \), where is the focus?
   c. If the focus is (2, -3) and the directrix is \( x = -1 \), where is the vertex?

2. Find the vertex of each parabola and state whether the parabola opens upward, downward, to the right, or to the left. Also give the equation of the axis of symmetry.
   a. \( y = x^2 - 5 \)
   b. \( y = -4x^2 \)
   c. \( x = 2y^2 + 1 \)
   d. \( x = -(y - 3)^2 \)
   e. \( y + 2 = -(x + 1)^2 \)
   f. \( \left( \frac{y - 4}{2} \right)^2 = \frac{x + 5}{4} \)

3. Write an equation in standard form for each parabola.
   a. 
   b. 
   c.
Lesson 8.4 • Hyperbolas

1. Write an equation in standard form for each graph.
   a. 
   b. 

2. Graph each hyperbola. Include the asymptotes and the foci. Write
   the equations of the asymptotes, and give the exact coordinates of the
   vertices and the foci.
   a. $y^2 - x^2 = 1$
   b. $\left(\frac{x-1}{3}\right)^2 - \left(\frac{y}{5}\right)^2 = 1$

3. Identify each path described as an ellipse or a hyperbola. Then write
   the equation in standard form for each path.
   a. A point moves in a plane so that the difference of its distances
      from the points $(0, 5)$ and $(0, -5)$ is always 8 units.
   b. A point moves in a plane so that the sum of its distances from the
      points $(-4, 0)$ and $(4, 0)$ is always 10 units.
Lesson 8.5 • The General Quadratic

1. Identify the graph of each equation as a circle, an ellipse, a parabola, or a hyperbola. Then rewrite each equation in the general quadratic form, \(Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0\). (Include all coefficients.)
   a. \((x - 7)^2 + (y + 2)^2 = 25\)
   b. \(\frac{(x - 4)^2}{9} - \frac{(y - 3)^2}{16} = 1\)
   c. \(0.5(x - 4)^2 = 6.5(y - 2)\)

2. Convert each equation to the standard form of a conic section.
   a. \(y^2 + 6y - x = 0\)
   b. \(x^2 - y^2 - 6x + 13 = 0\)
   c. \(x^2 + y^2 - 12x + 10y + 45 = 0\)
   d. \(16x^2 + 25y^2 - 32x + 100y - 284 = 0\)

3. Name the shape described by each equation in Exercise 2. Give the vertex of each parabola and the center of each circle, ellipse, and hyperbola.

4. Solve each equation for \(y\). You may need to use the quadratic formula.
   a. \(4x^2 + 9y^2 - 36 = 0\)
   b. \(x^2 + 2x - 4y^2 + 20 = 0\)
   c. \(3x^2 + 4y^2 - 5y + 2 = 0\)
   d. \(2x^2 + 3y^2 - 6x + 8y - 1 = 0\)

5. Solve each system of equations algebraically, using the substitution method or the elimination method.
   a. \(\begin{cases} y = x^2 - 4 \\ y = 2x - 1 \end{cases}\)
   b. \(\begin{cases} y = -x^2 + 5 \\ y = (x - 3)^2 \end{cases}\)
   c. \(\begin{cases} 2x^2 + 3y^2 - 5 = 0 \\ 3x^2 - 4y^2 + 1 = 0 \end{cases}\)
Lesson 8.6 • Introduction to Rational Functions

1. Write an equation and graph each transformation of the parent function \( f(x) = \frac{1}{x} \).
   a. Translate the graph left 3 units and down 4 units.
   b. Vertically dilate the graph by a scale factor of 3.

2. Write equations for the asymptotes of each hyperbola.
   a. \( y = \frac{2}{x} \)  
   b. \( y = \frac{1}{x + 3} \)  
   c. \( y = -\frac{3}{x} \)  
   d. \( y = \frac{1}{x} + 5 \)  
   e. \( y = \frac{1}{x - 2} - 6 \)  
   f. \( y = \frac{4}{x + 2} - 1 \)  

3. Solve.
   a. \( \frac{6}{x - 5} = -2 \)  
   b. \( \frac{4}{2x + 5} = \frac{1}{2} \)  
   c. \( -2 = \frac{2x + 14}{x - 1} \)  

4. Write a rational equation that can be used to solve each problem, using \( x \) as the variable. Then use your equation to solve the problem.
   a. A baseball player got 34 hits in his first 142 at-bats this season. How many consecutive hits must he get to bring his batting average up to .280?
   b. How much water must be added to 120 mL of a 35% alcohol solution to dilute it to a 25% alcohol solution?
Lesson 8.7 • Graphs of Rational Functions

1. Rewrite each rational expression in factored form.
   a. \( \frac{x^2 - 5x - 6}{x^2 - 25} \)
   b. \( \frac{x^2 - 16}{6x^2 - 7x - 3} \)
   c. \( \frac{9x^2 - 1}{2x^3 - x^2 - 3x} \)

2. Rewrite each expression in fractional form (as the quotient of two polynomials).
   a. \( \frac{2}{x} + 3 \)
   b. \( 4 + \frac{2x - 7}{x + 5} \)
   c. \( \frac{5x - 7}{x + 3} - 4 \)

3. Find all vertical and horizontal asymptotes of the graph of each rational function.
   a. \( f(x) = -\frac{1}{x^2} \)
   b. \( f(x) = \frac{3}{(x - 2)^2} \)
   c. \( f(x) = \frac{x^2 + x + 1}{x^2 - 4} \)

4. Find all vertical and slant asymptotes of the graph of each rational function.
   a. \( f(x) = \frac{x^2 + 1}{x} \)
   b. \( f(x) = \frac{x^3}{x^2 - 4} \)
   c. \( f(x) = \frac{9 - x^2}{2 + x} \)

5. Give the coordinates of all holes in the graph of each rational function.
   a. \( f(x) = \frac{x - 3}{3 - x} \)
   b. \( f(x) = \frac{2x + 6}{x + 3} \)
   c. \( f(x) = \frac{x^2 - 3x - 10}{x + 2} \)
Lesson 8.8 • Operations with Rational Expressions

1. Factor the numerator and denominator of each expression completely and reduce common factors.
   a. \( \frac{x^2 - 4x}{x^2 - x - 12} \)  
   b. \( \frac{x^2 - 49}{x^2 + 14x + 49} \)  
   c. \( \frac{2x^2 - 10x}{3x^3 - 11x - 20} \)  
   d. \( \frac{4x^2 - 1}{6x^2 - x - 2} \)  
   e. \( \frac{9x^2 - 30x + 25}{9x^2 - 12x - 5} \)  
   f. \( \frac{4x^2 + 21x + 5}{5x^2 + 23x - 10} \)  

2. Find the least common denominator for each pair of rational expressions. Don’t forget to factor the denominators completely first.
   a. \( \frac{3}{(x + 4)(x + 2)} - \frac{5}{(x + 4)(x - 5)} \)  
   b. \( \frac{3x}{x^2 - 16} - \frac{2x}{x^2 + 5x + 4} \)  
   c. \( \frac{2x - 1}{x^2 - 4x + 4} - \frac{3x^3}{x^2 - 6x + 8} \)  
   d. \( \frac{x + 3}{x^2 - 7x - 8} - \frac{x - 5}{2x^2 + x} \)  

3. Add, subtract, multiply, or divide as indicated. Reduce any common factors.
   a. \( \frac{3}{(x + 2)(x - 1)} + \frac{5}{(x + 1)(x - 1)} \)  
   b. \( \frac{4}{x^2 - 49} - \frac{x}{(x + 7)(x - 1)} \)  
   c. \( \frac{x^2 + 2x - 15}{2x^2 + 9x - 5} \times \frac{4x^2 - 1}{2x^2 - 5x - 3} \)  
   d. \( \frac{9x^2 + 6x}{2x - 1} \div \frac{6x^2 + x - 2}{4x^2 - 4x + 1} \)  

4. Rewrite each fraction as a single rational expression.
   a. \( \frac{2x - 1}{2x^2 + 3x - 2} \)  
   b. \( \frac{x^2 - 9}{x^2 - 2x - 3} \times \frac{x^2 + 6x + 9}{x + 1} \)  
   c. \( \frac{1}{x - 2} + \frac{1}{x + 2} \)  

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Lesson 9.1 • Arithmetic Series

1. List the first six terms of each arithmetic sequence and identify the common difference.
   a. \( u_1 = 5 \)
      \[ u_n = u_{n-1} + 6 \quad \text{where} \quad n \geq 2 \]
   b. \( a_1 = 7.8 \)
      \[ a_n = a_{n-1} - 2.3 \quad \text{where} \quad n \geq 2 \]

2. Write each expression as a sum of terms and calculate the sum.
   a. \[ \sum_{n=1}^{3} (n - 5) \]
   b. \[ \sum_{n=1}^{4} (3n - 7) \]
   c. \[ \sum_{n=1}^{5} (2n^2 + 5) \]

3. Find the indicated values.
   a. \( u_{18} \) if \( u_n = \frac{2}{3}n + \frac{3}{4} \)
   b. \( S_{10} \) if \( u_n = 3n - 6 \)
   c. \[ \sum_{n=51}^{100} (9n - 81) \]

4. There are 22 rows of seats in a high school auditorium. There are 17 seats in the front row, and each of the other rows has two more seats than the row directly in front of it.
   a. List the first six terms of the sequence that describes the number of seats in each row, starting with the front row.
   b. Write a recursive formula for this sequence.
   c. Write an explicit formula for the number of seats in row \( n \).
   d. The rows are identified with letters to help people who attend performances in the auditorium find their seats. If the front row is row A, how many seats are there in row M?
   e. How many seats are there in the back row?
   f. How many seats are there in the auditorium?
Lesson 9.2 • Infinite Geometric Series

1. Consider the repeating decimal 0.393939..., or 0.39.
   a. Express the decimal as the sum of terms of an infinite geometric sequence.
   b. Identify the first term and common ratio of the sequence.
   c. Express the infinite sum as a ratio of integers in lowest terms.

2. Find the common ratio for each geometric sequence. State whether each series is convergent or not convergent. If the series is convergent, find the sum.
   a. $4 + 3.2 + 2.56 + 2.048 + \cdots$
   b. $10 - 5 + 2.5 - 1.25 + \cdots$
   c. $-2 + 2.2 - 2.42 + 2.662 - \cdots$
   d. $13 + 1.3 + 0.13 + \cdots$

3. Evaluate each sum.
   a. $\sum_{n=1}^{\infty} -3\left(\frac{3}{4}\right)^{n-1}$
   b. $\sum_{n=1}^{\infty} 1.1(0.5)^{n-1}$
   c. $\sum_{n=1}^{\infty} 12\left(-\frac{1}{4}\right)^{n-1}$

4. The first term of a geometric sequence is 1 and the sum of the series is 3. Find the common ratio and list the first five terms of the sequence.

5. The common ratio of a geometric sequence is $-0.6$ and the sum of the series is 44. Find the first term and list the first four terms of the sequence.

6. A pendulum bob swings through a 50 cm arc on its first swing. For each swing after the first, it swings only 78% as far as on the previous swing. How far will the bob swing altogether before coming to a complete stop?
Lesson 9.3 • Partial Sums of Geometric Series

1. For each partial sum equation, identify the first term, the common ratio, and the number of terms.
   a. \[
   \frac{15}{1 - 0.6} - \frac{15}{1 - 0.6} \cdot 0.6^5 = 34.584
   \]
   b. \[
   \frac{30 - 0.1171875}{1 - 0.5} = 59.765625
   \]

2. Consider the geometric sequence 187.5, 75, 30, 12, . . .
   a. What is the tenth term?
   b. Which term is the first one smaller than 1?
   c. Find \(u_n\).
   d. Find \(S_n\).

3. Find the first term and the common ratio or common difference of each series. Then find the partial sum.
   a. \(5 + 6.2 + 7.4 + \cdots + 17\)
   b. \(150 - 30 + 6 - 1.2 + \cdots + 0.000384\)
   c. \[
   \sum_{n=1}^{15} 12.5(1.1)^{n-1}
   \]
   d. \[
   \sum_{n=1}^{50} (72 - 3.5n)
   \]

4. Find the missing values.
   a. \(u_1 = 4, r = 3, S_{10} = \) \_
   b. \(u_1 = 2, r = 0.8, S_{\text{\_
}} = 6.7232\)
   c. \(u_1 = \text{\_
}, r = 1.1, S_6 = 92.58732\)
   d. \(u_1 = 10, r = \text{\_
}, S_{12} = 19.99511719\)

5. Suppose that you rent an apartment for $750 a month. Each year, your landlord raises the rent by 5%.
   a. If you make a list of your monthly rent over several years, does this form an arithmetic or geometric sequence? If the sequence is arithmetic, give the common difference; if it is geometric, give the common ratio.
   b. To the nearest dollar, what is your monthly rent during the fifth year you rent the apartment?
   c. To the nearest ten dollars, what is the total amount of rent that you paid during the first five years you rented the apartment?
Lesson 10.1 • Randomness and Probability

Name ___________________________  Period ____________  Date ________________

1. A national survey was taken measuring the highest level of educational achievement among adults age 30 and over. Express each probability to the nearest 0.001.

<table>
<thead>
<tr>
<th>Highest level of education</th>
<th>Women</th>
<th>Men</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>8th grade or less</td>
<td>35</td>
<td>46</td>
<td>81</td>
</tr>
<tr>
<td>High school graduate</td>
<td>232</td>
<td>305</td>
<td>537</td>
</tr>
<tr>
<td>Some college</td>
<td>419</td>
<td>374</td>
<td>793</td>
</tr>
<tr>
<td>Bachelor's degree</td>
<td>539</td>
<td>463</td>
<td>1002</td>
</tr>
<tr>
<td>Graduate or professional degree</td>
<td>377</td>
<td>382</td>
<td>759</td>
</tr>
<tr>
<td>Total</td>
<td>1602</td>
<td>1570</td>
<td>3172</td>
</tr>
</tbody>
</table>

a. What is the probability that a randomly chosen person from the survey group is a man?

b. What is the probability that the highest level of education completed by a randomly chosen person from the survey group is a bachelor’s degree?

c. What is the probability that a randomly chosen woman has earned a bachelor’s or graduate degree?

d. Are the probabilities in 1a–c experimental or theoretical?

2. Suppose that a bag contains five green marbles, three blue marbles, six yellow marbles, and four white marbles. Maria shakes up the bag to mix the marbles and then draws one marble out of the bag.

a. What is the probability that the marble Maria draws is blue?

b. What is the probability that the marble is green or yellow?

c. What is the probability that the marble is neither blue nor yellow?

D. Are the probabilities in 2a–c experimental or theoretical?
Lesson 10.2 • Counting Outcomes and Tree Diagrams

1. Find the probability of each branch or path, a–g, in the tree diagram below.

   a
   b
   f
   c
   0.6392
   d
   0.0256
   e
   g
   0.32

2. Draw a tree diagram that shows all possible equally likely outcomes if a penny is tossed once and then a six-sided die is rolled once. Then use your diagram to find each probability.

   a. What is the probability of tossing a head and rolling a 5?
   b. What is the probability of tossing a tail and rolling an even number?

3. Liam draws one playing card from a 52-card deck and places it on the table. Then he draws a second card and places it to the right of the first card. What is the probability that both cards are black?

4. Three friends try out for sports teams at their high school. Gladys tries out for the lacrosse team and has a 40% chance of success (making the team). Becky tries out for the synchronized swim team and has a 30% chance of success. Serita tries out for the tennis team and has a 25% chance of success. Use the tree diagram to find each probability.

   a. What is the probability that all three girls will make their teams?
   b. What is the probability that exactly one of the girls will be successful?
Lesson 10.3 • Mutually Exclusive Events and Venn Diagrams

1. Refer to the Venn diagram, which gives probabilities related to the two events “plays the piano” and “plays the violin.” These probabilities apply to the students at Riverway Middle School, which has 800 students.

   a. What is the probability that a randomly chosen student at Riverway plays the piano?

   b. How many students at Riverway play both instruments?

2. Refer to the Venn diagram in Exercise 1. Find the probability of each event indicated. N represents the event that a student plays the piano, and V represents the event that a student plays the violin.

   a. \(P(\text{not } N)\)
   b. \(P(N \text{ or } V)\)
   c. \(P(\text{not } N \text{ and not } V)\)

3. For a class project, Diana surveys 300 students at her high school about the entertainment equipment (MP3 players, gaming systems, and DVD players) they have in their homes. She gathers the following information.

   187 homes had gaming systems and 141 homes had DVD players.
   19 homes had no entertainment equipment, whereas 12 homes had DVD players only.
   81 homes had gaming systems and MP3 players, but not DVD players.
   11 homes had gaming systems and DVD players, but not MP3 players.
   43 homes had MP3 players and DVD players, but not gaming systems.

   a. Complete the Venn diagram using probabilities.

   b. What is the probability that a student’s home has a MP3 player, but neither a gaming system nor a DVD player?

   c. What is the probability that a student’s home has all three pieces of equipment?
Lesson 10.4 • Random Variables and Expected Value

1. Determine whether each number described below comes from a random variable, a discrete random variable, or a geometric random variable. (List all the terms that apply.)
   
   a. The number of times that you get heads if you toss a nickel 100 times

   b. The height of a building, measured to the nearest tenth of a meter

   c. The number of times you roll a pair of dice until you get “doubles”

2. You know that 35% of the students in your high school own laptops. Suppose you stop random students who enter the school on a particular day and ask them if they own laptops.

   a. What is the probability that the fourth student you ask owns a laptop?

   b. What is the probability that you will find your first laptop owner within the first four students you ask?

3. Suppose that you enter a contest that promises to award these prizes:
   
   1 first prize: $10,000
   100 third prizes: $100 each
   50 second prizes: $1,000 each
   1,000 fourth prizes: $20 each

   One entry is allowed per person, and 50,000 people enter the contest. Let $x$ represent the random variable. Its values are the possible amounts (in dollars) that you may win.

   a. Complete the table to show the probabilities of each possible value of $x$. (Write each probability as an exact decimal.)

   $\begin{array}{c|c|c|c|c|c}
   x_i & 10,000 & 1,000 & 100 & 20 & 0 \\
   \hline
   P(x_i) & \text{[ ]} & \text{[ ]} & \text{[ ]} & \text{[ ]} & \text{[ ]} \\
   \end{array}$

   b. How many people who enter the contest will not win any prize?

   c. If there is no cost to enter the contest, what are your expected winnings?
Lesson 10.5 • Permutations and Probability

1. Give the number of possible arrangements or selections for each situation.
   a. Arrangements of six poetry books on a shelf

   b. License plates with two letters followed by four digits

   c. Outfits made up of a shirt, a pair of slacks, and a sweater selected from five shirts, four pairs of slacks, and three sweaters

   d. Seven-digit telephone numbers, if the first digit cannot be 0

   e. Three-digit integers that are multiples of 5 and have no repeated digits

2. Evaluate each expression. (Some answers will be in terms of \( n \).)
   a. \( \frac{15!}{14!} \)
   b. \( \frac{21!}{19!} \)
   c. \( \frac{(n + 1)!}{(n - 1)!} \)

   d. \( 10P_2 \)
   e. \( 15P_4 \)
   f. \( nP_{n-3} \)

3. In Ms. Scarpino’s math class, there are six desks in each row. On the first day of the semester, she tells her students that they may sit anywhere they want, but that they must sit in the same row every day.
   a. If the first row is completely filled, in how many different ways can the students who have chosen to sit there be seated?

   b. What is the probability that the students in the front row will be seated in alphabetical order?

   c. What is the probability that among the students in the front row, the tallest student will sit in the chair farthest to the right?
Lesson 10.6 • Combinations and Probability

1. Evaluate each expression without using a calculator.
   a. \( \frac{5!}{3!2!} \)
   b. \( \frac{8!}{3!5!} \)
   c. \( \frac{25!}{0!25!} \)

2. Evaluate each expression.
   a. \( \binom{6}{3} \)
   b. \( \binom{8}{7} \)
   c. \( \binom{20}{1} \)
   d. \( \binom{199}{199} \)

3. Find the number of ways of making each choice.
   a. Selecting three days out of a week if exactly two of them must be weekdays

   b. Selecting a 4-member committee from a 20-member club

   c. Selecting a 4-member committee from a 20-member club if there are 12 women and 8 men in the club and the committee must include 2 men and 2 women

4. There are 10 fourth graders, 12 fifth graders, and 8 sixth graders in a Girl Scout troop. Mrs. Sullivan, the troop leader, needs five girls to serve on the troop’s camping committee. To make the selection fair, she lets the girls draw names out of a hat to fill the six places on the committee.
   a. How many different committees are possible?

   b. What is the probability that Lisa Brownell, one of the sixth-grade scouts, will be on the committee?

   c. What is the probability that the committee will be made up of 2 fourth graders, 2 fifth graders, and 1 sixth grader?
Lesson 10.7 • The Binomial Theorem and Pascal’s Triangle

Name ___________________________   Period _________   Date ________________

1. Find the indicated term of each binomial expansion.
   a. \((2x - y)^3\); 3rd term
   b. \((m - n)^5\); 4th term
   c. \((2a + 3b)^5\); 6th term

2. Find each of the following probabilities for a family with six children. Assume that having a boy and having a girl are equally likely. (Write each probability as a fraction and also as an exact decimal.)
   a. What is the probability that the family will have five girls and one boy?

   b. What is the probability that there will be at least three girls?

3. The Bright Company manufactures lightbulbs. Over time, the Quality Assurance Department has determined that 2\% of all bulbs manufactured in the Bright factory are defective. Determine each probability for a random sample of 20 Bright lightbulbs. (Give your answers as decimals rounded to the nearest thousandth.)
   a. What is the probability that there will be no defective bulbs in the sample?

   b. What is the probability that there will be exactly two defective bulbs in the sample?

   c. What is the probability that there will be no more than two defective bulbs in the sample?

4. Suppose that the probability of success is 0.79. What probability question is answered by the expression \[
\sum_{r=10}^{30} C_{r}(0.79)^r(0.21)^{30-r} \]?
Lesson 11.1 • Experimental Design

For Exercises 1 and 2, use each of the three scenarios to answer the questions.

a. The Glass Pane Company calls numbers on a Monday that were randomly chosen from the telephone book and asks the person who answers the phone, “In what year were the windows in your home installed?”

b. Mario wanted to determine whether temperature affects the level of animal activity. At a park near his house one day he recorded the number of animals he saw between 1:00 P.M. and 2:00 P.M. when it was 85°F. Then at the same park on another day he recorded the number of animals he saw between 1:00 P.M. and 2:00 P.M. when it was 55°F.

c. Sienna wants to know if the citizens of her town think the price of a movie ticket is fair and asks every other person standing in the ticket line, “Do you think movie tickets are overpriced?”

1. Identify which type of data collection was used.

2. Identify at least one source of bias in each study design in each scenario. Explain your reasoning.

3. A radio station announcer asked listeners to call in and four of the five listeners who responded were under 20 years old. She then stated that over 75% of her listeners are under the age of 20. Is this statistic reliable? Explain.

4. Scott thinks eating fruit before a test will increase a person’s score on the test. Just before a test in his class, he randomly gives fruit to half of his classmates, and those students eat the fruit. The rest of the students do not eat fruit. He then compares the scores after the test.

   a. Identify which type of data collection was used.

   b. Identify the treatment and state how it was assigned.
Lesson 11.2 • Probability Distributions

1. A random-number generator selects a number between 0 and 5 according to the continuous probability distribution shown in the graph.
   Find the probability that the random number will be
   a. Less than 2
   b. Between 2 and 4
   c. Greater than 3
   d. Between 2.5 and 5

2. The graph represents a probability distribution.
   a. Find the value of the \(y\)-scale so that the area is 1.
   b. Find the probability that a randomly chosen value will be greater than 5.
   c. Find the mode.
   d. Find the median to the nearest tenth.
   e. Find the mean to the nearest tenth.

3. The graph represents a continuous probability distribution.
   The boundary of this graph is an isosceles trapezoid.
   a. Find the height of the trapezoid.
   b. Find the mean and the median.
   c. Does this distribution have a mode? Explain.
Lesson 11.3 • Normal Distributions

1. Use the graphs to estimate the mean and standard deviation of each distribution.

   a. 
   
   b. 

2. Estimate the equation of each graph in Exercise 1.

3. From each equation, find or estimate the mean and standard deviation.
   
   a. \[ y = \frac{1}{8\sqrt{2\pi}} \left(\sqrt{e}\right)^{-\left(\frac{(x-55)^2}{8}\right)} \]
   
   b. \[ y = \frac{0.4}{0.75} \left(0.60653\right)^{-\left(\frac{(x-4.8)/0.75}{2}\right)} \]

4. The weights of 1000 children were recorded on their first birthdays. The weights are normally distributed with mean 10.3 kg and standard deviation 1.6 kg.
   
   a. What is the probability that a randomly selected child will weigh less than 10 kg?

   b. What is the probability that a randomly selected child will weigh between 8.7 kg and 11.9 kg?

   c. How many of the 1000 children would you expect to weigh between 8.3 kg and 12.3 kg?

   d. How many of the 1000 children would you expect to weigh more than 8.7 kg?
Lesson 11.4 • z-Values and Confidence Intervals

1. The heights of a group of 500 women are normally distributed with mean 65 inches and standard deviation 2.2 inches. Find the height for each of these z-values to the nearest tenth of an inch.
   a. $z = 2$
   b. $z = 0.5$
   c. $z = -3.4$

2. The mean commuting time for a resident of a certain metropolitan area is 38 minutes, with a standard deviation of 10 minutes. Assume that commuting times for this area are normally distributed.
   a. Find the z-value for a 23-minute commute.
   b. Find the z-value for a 60-minute commute.
   c. What is the probability that a commute for a randomly chosen resident will be between 28 minutes and 58 minutes?

3. A sample has mean 52.6 and standard deviation 6.4. Find each confidence interval. Assume $n = 25$ in each case.
   a. 68% confidence interval
   b. 90% confidence interval
   c. 99% confidence interval
   d. 99.5% confidence interval
Lesson 11.5 • Bivariate Data and Correlation

1. Complete this table. Then answer 1a–d to calculate the correlation coefficient.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>x − ̅x</th>
<th>y − ̅y</th>
<th>(x − ̅x)(y − ̅y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
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<td>8</td>
<td>13</td>
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<td>10</td>
<td>15</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. What are ̅x and ̅y?

b. What is the sum of the values for (x − ̅x)(y − ̅y)?

c. What are sx and sy?

d. Calculate \( r = \frac{\sum (x - ̅x)(y - ̅y)}{sx sy (n - 1)} \).

e. What does this value of r tell you about the data?

2. For each research finding, decide whether the relationship is causation, correlation, or both. If it is only correlation, name a possible lurking variable that may be the cause of the results.

a. A pharmaceutical company made a television commercial based on the finding that people who took their daily vitamins lived longer. Do vitamins extend life?

b. A high school counselor observed that students who take a psychology course are never in the school band. Does this mean that students who are interested in psychology have less musical ability than other students do?
Lesson 11.6 • The Least Squares Line

1. The data at right show the population of Lenoxville at five-year intervals. Use the data to calculate each specified value.
   a. \( \bar{x} \)  
   b. \( \bar{y} \)  
   c. \( s_x \)  
   d. \( s_y \)  
   e. \( r \)

<table>
<thead>
<tr>
<th>Year</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>15,420</td>
</tr>
<tr>
<td>1990</td>
<td>14,860</td>
</tr>
<tr>
<td>1995</td>
<td>14,215</td>
</tr>
<tr>
<td>2000</td>
<td>13,390</td>
</tr>
<tr>
<td>2005</td>
<td>12,625</td>
</tr>
</tbody>
</table>

2. Use the given statistics to calculate the least squares line for each data set described.
   a. \( \bar{x} = 3, s_x = 0.25, \bar{y} = 6.5, s_y = 1.5, r = -0.8 \)

   \( b. \bar{x} = 28.4, s_x = 2.5, \bar{y} = 8.3, s_y = 0.9, r = 0.98 \)

3. Use the data from Exercise 1 and the least squares equation 
   \( \hat{y} = 295,796 - 141.2x \) to calculate
   a. The sum of the residuals
   b. The sum of the squares of the residuals
   c. The root mean square error

4. The table shows the percentage of U.S. television-owning households that had basic cable TV service in certain years.

<table>
<thead>
<tr>
<th>Year</th>
<th>2000</th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Percent</td>
<td>77.9</td>
<td>79.8</td>
<td>83.8</td>
<td>82.9</td>
<td>85.3</td>
<td>85.7</td>
<td>86.2</td>
</tr>
</tbody>
</table>

(The World Almanac and Book of Facts 2007)

   a. Find the equation of the least squares line for these data. Let \( x = 0 \) represent the year 1990.
   b. What are the real-world meanings of the slope and the \( y \)-intercept?
Lesson 12.1 • Right Triangle Trigonometry

1. For each of the following right triangles, find the values of \( \sin A \), \( \cos A \), \( \tan A \), \( \sin B \), \( \cos B \), and \( \tan B \). (Write your answers as fractions in lowest terms.)

   a. 
   
   ![Triangle A](image)
   
   b. 
   
   ![Triangle B](image)

2. Draw a right triangle for each problem. Label the sides and angles with the given measures, then solve to find the unknown value. Round your answers to the nearest tenth.

   a. \( \cos 27^\circ = \frac{r}{8.5} \)
   b. \( \tan^{-1} \left( \frac{9}{10} \right) = S \)
   c. \( \cos 52^\circ = \frac{z - 3}{x} \)

3. For each triangle, write an equation to calculate the labeled measure. Then, solve the equation.

   a. 
   
   ![Triangle C](image)
   
   b. 
   
   ![Triangle D](image)
   
   c. 
   
   ![Triangle E](image)
Lesson 12.2 • The Law of Sines

1. Solve each equation for \( b \). Give an exact answer and an approximate answer. Round to the nearest tenth.
   \[
   \begin{align*}
   a. \quad \frac{\sin 75^\circ}{9} &= \frac{\sin 20^\circ}{b} \\
   b. \quad \frac{\sin 95^\circ}{b} &= \frac{\sin 45^\circ}{6.2} \\
   c. \quad \frac{\sin 32.4^\circ}{12} &= \frac{\sin 120.5^\circ}{b}
   \end{align*}
   \]

2. Find the unknown angle measures and side lengths. Round to the nearest tenth.
   \[
   \begin{align*}
   a. \quad \angle B &= 64.8^\circ, \quad \angle A = 42.5^\circ, \quad \angle C = 12.8^\circ, \\
   \quad b &= 5.4 \text{ cm}, \quad a = 12.5 \text{ cm}, \quad c = 11.9 \text{ cm} \\
   b. \quad \angle A &= 33^\circ, \quad \angle B = 122^\circ, \quad \angle C = 25^\circ, \\
   \quad a &= 18.5 \text{ mm}, \quad b = 25 \text{ mm}, \quad c = 32 \text{ mm}
   \end{align*}
   \]

3. Determine the number of triangles with the given parts. (Do not find the missing side lengths and angle measures.)
   \[
   \begin{align*}
   a. \quad B &= 50^\circ, \quad b = 22.6 \text{ cm}, \quad c = 27.2 \text{ cm} \\
   b. \quad A &= 68^\circ, \quad a = 25 \text{ cm}, \quad c = 32 \text{ cm}
   \end{align*}
   \]

4. A ship is sailing due east. At a certain point, the captain observes a lighthouse at an angle of 32° clockwise from north. After the ship sails 28.0 km farther, the same lighthouse is at an angle of 310° clockwise from north.
   a. Draw a diagram at right to illustrate this situation. Let \( A \) be the point where the first lighthouse observation is made, \( B \) be the point where the second observation is made, and \( C \) be the location of the lighthouse. Include all given information in your diagram.
   b. Use the Law of Sines to find the distance between the lighthouse and the ship at the time of the first observation.
   c. Use the Law of Sines to find the distance between the lighthouse and the ship at the time of the second observation.
Lesson 12.3 • The Law of Cosines

Name ___________________________   Period _______   Date ______________

1. Solve for \( b \) and \( C \). Assume \( b \) is positive.
   a. \( b^2 = 14^2 + 29^2 - 2(14)(29) \cos 123.5^\circ \)
   b. \( 3.8^2 = 4.0^2 + 5.1^2 - 2(4.0)(5.1) \cos C \)

2. Use the given values to write an equation for the unknown measure.
   Then solve the equation. Give an exact answer and an approximate answer. Round to the nearest tenth.
   a. \( b = 9 \)  \( c = 12 \)  \( A = 110^\circ \)
   b. \( a = 5 \)  \( c = 4 \)  \( B \)
   c. \( a = 3.5 \)  \( b = 6.9 \)  \( C = 82.5^\circ \)

3. State whether you would use the Law of Sines or the Law of Cosines to solve each problem.
   a. Given the measures of two angles of a triangle and the length of one of the sides that is not between them, find the length of one of the other two sides.
   b. Given the lengths of all three sides of a triangle, find the measure of the smallest angle.

4. Find unknown angle measures and side lengths. Round to the nearest tenth.
   a. \( 148^\circ \) \( 12.8 \text{ cm} \) \( 5.2 \text{ cm} \)
   b. \( 6.9 \text{ cm} \) \( 3.8 \text{ cm} \) \( 7.5 \text{ cm} \)

5. Use the Law of Cosines to find the measure of the largest angle of a triangular garden whose sides measure 12.5 ft, 19.8 ft, and 15.7 ft.
Lesson 12.4 • Extending Trigonometry

1. Sketch each angle in the coordinate plane. Then, find the measure of the reference angle for each angle.
   a. 150°
   b. −30°
   c. 200°

2. Find the trigonometric value requested for each angle.
   a. \( \cos \theta \)
   b. \( \tan \alpha \)

3. Without using a calculator, determine whether each value is positive or negative.
   a. \( \tan 100° \)
   b. \( \sin 195° \)
   c. \( \cos 70° \)
   d. \( \sin −100° \)
   e. \( \tan 260° \)
   f. \( \cos 260° \)
Lesson 12.5 • Introduction to Vectors

1. Let \( \mathbf{a} = \langle 2, 3 \rangle \), \( \mathbf{b} = \langle -3, -2 \rangle \), \( \mathbf{c} = \langle 0, 1 \rangle \), and \( \mathbf{d} = \langle -1, 5 \rangle \). Evaluate each of the following expressions.
   a. \( \mathbf{a} + \mathbf{b} \)  
   b. \( \mathbf{c} - \mathbf{b} \)
   c. \( |\mathbf{d}| \)  
   d. \( 2 \mathbf{c} \)

2. Write each vector in rectangular form.
   a. \( \langle 2 \angle 45^\circ \rangle \)  
   b. \( \langle 2 \angle 90^\circ \rangle \)  
   c. \( \langle 5 \angle 270^\circ \rangle \)

3. Write each vector in polar form. Round each angle measure to the nearest degree.
   a. \( \langle -2, 0 \rangle \)  
   b. \( \langle 3, 4 \rangle \)  
   c. \( \langle -2, -1 \rangle \)

4. A boat leaves a dock and travels 20 miles on a bearing of 58°. It then turns and travels 64 miles on a bearing of 352° to reach a second dock.
   a. Draw a sketch of the situation.
   b. Find the ship’s distance and bearing from the initial dock. Round your answers to the nearest tenth.
Lesson 12.6 • Parametric Equations

1. Create a table for each pair of parametric equations with the given values of $t$.
   a. $x = t + 5$
      
      $y = t^2 + 1$
      
      $t = \{-2, -1, 0, 1, 2\}$
   
   b. $x = |t - 3|$
      
      $y = |t + 3|$
      
      $t = \{-4, -2, 0, 2, 4\}$

2. Write a single equation (using only $x$ and $y$) that is equivalent to each pair of parametric equations. (Each equation should express $y$ in terms of $x$.)
   a. $x = t$
      
      $y = 3t^2 + 2t - 1$
   
   b. $x = t^2$
      
      $y = 3t - 2$
   
   c. $x = \frac{t^2 + 4}{2}$
      
      $y = 3t - 5$

3. Use the graphs of $x = f(t)$ and $y = g(t)$ to create a graph of $y$ as a function of $x$.

4. Hayden rolls a ball off the edge of the roof of a 75 ft tall building at an initial velocity of 6.5 ft/s.
   a. Write parametric equations to simulate this motion.

   b. What equation can you solve to determine when the ball hits the ground?

   c. When and where does the ball hit the ground? (Round to the nearest hundredth.)
Lesson 13.1 • Defining the Circular Functions

1. Find the exact value of each expression.
   a. \( \cos 45^\circ \)  
   b. \( \sin(-30^\circ) \)  
   c. \( \cos 240^\circ \)  
   d. \( \sin 360^\circ \)

2. Use your calculator to find each value, approximated to four decimal places. Then draw a diagram in a unit circle to represent the value. Name each reference angle.
   a. \( \sin 37^\circ \)  
   b. \( \cos 115^\circ \)  
   c. \( \sin(-21^\circ) \)

3. Determine whether each function whose graph is shown below is periodic or not periodic. For each periodic function, identify the period.
   a.
   ![Graph A]
   b.
   ![Graph B]

4. Identify an angle \( \theta \) that is coterminal with the given angle. Use domain \( 0^\circ \leq \theta \leq 360^\circ \).
   a. \( -42^\circ \)  
   b. \( 415^\circ \)  
   c. \( 913^\circ \)  
   d. \( -294^\circ \)

5. Let \( \theta \) represent the angle between the x-axis and the ray with endpoint \((0, 0)\) passing through \((-3, 3)\). Find \( \sin \theta \) and \( \cos \theta \).
Lesson 13.2 • Radian Measure

1. Convert between radians and degrees. Give exact answers.
   a. \( \frac{5\pi}{4} \)  
   b. \( 15^\circ \)  
   c. \( 330^\circ \)  
   d. \( -\frac{2\pi}{3} \)
   e. \( -140^\circ \)  
   f. \( 780^\circ \)  
   g. \( -\frac{11\pi}{6} \)  
   h. \( \frac{17\pi}{15} \)

2. Find the length of the intercepted arc for each central angle.
   a. \( r = 8 \) and \( \theta = \frac{5\pi}{4} \)  
   b. \( r = 5.4 \) and \( \theta = 2.5 \)  
   c. \( d = 3 \) and \( \theta = \frac{\pi}{12} \)

3. Solve for \( \theta \).
   a. \( \sin \theta = \frac{\sqrt{3}}{2} \) and \( 90^\circ \leq \theta \leq 180^\circ \)  
   b. \( \sin \theta = -1 \) and \( 0^\circ \leq \theta \leq 360^\circ \)  
   c. \( \cos \theta = \frac{1}{2} \) and \( \pi \leq \theta \leq 2\pi \)  
   d. \( \frac{\sin \theta}{\cos \theta} = \frac{1}{\sqrt{3}} \) and \( 0 \leq \theta \leq \frac{\pi}{2} \)

4. The minute hand on a watch is 85 mm long. Round your answers in 4a and b to the nearest tenth, and in 4c to the nearest thousandth.
   a. What is the distance the tip of the minute hand travels, in mm?
   b. At what speed is the tip moving, in mm/min?
   c. What is the angular speed of the tip, in radians/min?
Lesson 13.3 • Graphing Trigonometric Functions

1. Write an equation for each sinusoid as a transformation of the graph of either \( y = \sin x \) or \( y = \cos x \). More than one answer is possible. Describe the amplitude, period, phase shift, and vertical shift of each graph.

   a. 
   
   b. 

2. Graph each function for the interval \( 0 \leq \theta \leq 2\pi \).

   a. \( y = 2 \sin x + 1 \)
   
   b. \( y = -\tan x \)
   
   c. \( y = 3 \cos \left( x + \frac{\pi}{4} \right) \)

3. Write an equation for each sinusoid with the given characteristics.

   a. A cosine curve with amplitude 2.5, period \( 2\pi \), and phase shift \( \frac{\pi}{4} \)

   b. A sine function with minimum value 2, maximum value 8, and one cycle starting at \( x = 0 \) and ending at \( x = \frac{3\pi}{2} \)
Lesson 13.4 • Inverses of Trigonometric Functions

1. Find the principal value of each expression to the nearest tenth of a degree and then to the nearest hundredth of a radian.
   a. \( \sin^{-1} 0.5976 \)  
   b. \( \cos^{-1}(-0.0315) \)  
   c. \( \cos^{-1}(0.8665) \)  
   d. \( \sin^{-1}(-0.6789) \)

2. Find all four values of \( x \) between \(-2\pi\) and \(2\pi\) that satisfy each equation.
   a. \( \sin x = \sin \frac{2\pi}{3} \)  
   b. \( \cos x = \cos \frac{5\pi}{12} \)  
   c. \( \sin x = \sin 1.25 \)  
   d. \( \cos x = \cos 0.73 \)  
   e. \( \cos x = \cos \frac{3\pi}{5} \)  
   f. \( \sin x = \sin \left( -\frac{5\pi}{6} \right) \)

3. Find values of \( x \) approximate to three decimal places that satisfy the conditions given.
   a. Find the first two positive solutions of \( \sin x = 0.9827 \).

   b. Find the first two positive solutions of \( \cos x = 0.7205 \).

4. Find the measure of the smallest angle of a triangle in which two of the sides have lengths 13 cm and 28 cm, and in which the angle opposite the 28 cm side measures 110°.
Lesson 13.5 • Modeling with Trigonometric Equations

1. Find all solutions for $0 \leq x < 2\pi$. Give exact values in radians.
   a. $\sin x = 1$
   b. $\cos x = 0$
   c. $\cos 3x = 0.5$
   d. $2\sin \left(\frac{1}{2}x\right) = 1$
   e. $\sin \frac{x}{2} = \frac{\sqrt{2}}{2}$
   f. $\cos \left(x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2}$

2. Find all solutions for $0 \leq x < 2\pi$, rounded to the nearest hundredth.
   a. $3 \cos(x + 0.4) = 2.6$
   b. $5 + 0.5 \sin 2x = 4.7$

3. Consider the graph of the function
   
   $h = 8.5 + 5 \sin \left[\frac{2\pi(t - 4)}{7}\right]$

   a. What is the vertical translation?
   b. What is the average value?
   c. What is the vertical scale factor?
   d. What is the minimum value?
   e. What is the maximum value?
   f. What is the amplitude?
   g. What is the horizontal scale factor?
   h. What is the period?
   i. What is the horizontal translation?
   j. What is the phase shift?

4. The number of hours of daylight on any day of the year in Philadelphia, Pennsylvania, is modeled using the equation
   
   $y = 12 + 2.4 \sin \left[\frac{2\pi(x - 80)}{365}\right]$

   where $x$ represents the day number (with January 1 as day 1). This equation assumes a 365-day year (not a leap year).
   a. Find the number of hours of daylight in Philadelphia on day 172, the longest day of the year (the summer solstice).
   b. Find the day numbers of the two days when the number of hours of daylight is closest to 13.
Lesson 13.6 • Fundamental Trigonometric Identities

1. Evaluate. Give exact values.
   a. \( \tan \frac{\pi}{3} \)      b. \( \cot \frac{5\pi}{6} \)      c. \( \sec \frac{\pi}{4} \)
   d. \( \csc \frac{4\pi}{3} \)      e. \( \cot \pi \)      f. \( \csc \frac{7\pi}{6} \)

2. Find another function that has the same graph as each function below. (More than one answer is possible.)
   a. \( y = \tan(x + \pi) \)       b. \( y = \sin(x - 2\pi) \)       c. \( y = -\csc(x - 2\pi) \)

3. Use trigonometric identities to rewrite each expression in a simplified form containing only sines and cosines, or as a single number.
   a. \( \tan \theta + \sec \theta \)       b. \( (\sec^2 \theta - \tan^2 \theta)\cos^2 \theta \)
   c. \( \cot \theta \sin^2 \theta - \tan \theta \cos^2 \theta \)       d. \( (\csc \theta + \cot \theta)(\csc \theta - \cot \theta) \)

4. Determine whether each equation is an identity or not an identity.
   a. \( \sin(A + \pi) = \cos A \)       b. \( \tan\left(A - \frac{\pi}{2}\right) = -\cot A \)
   c. \( \csc^2 A = \cot A(\tan A + \cot A) \)       d. \( \sec A \cot A = \csc A \)
Lesson 13.7 • Combining Trigonometric Functions

1. Use a graph or substitute values of A and B to decide whether each equation is an identity or not an identity.
   a. \( \cos 2A = 1 - 2\sin^2 A \)
   b. \( \sin(2\pi - A) = \sin A \)
   c. \( \tan 2A = \frac{\sin 2A}{\cos 2A} \)
   d. \( \cos(A - B) = \cos A \cos B - \sin A \sin B \)

2. Use identities from this lesson to derive an identity for \( \sin 3A \) in terms of \( \sin A \) and \( \cos A \). Show the steps you used to derive the identity.

3. Rewrite each expression with a single sine or cosine.
   a. \( \sin 3.2 \cos 2.5 - \cos 3.2 \sin 2.5 \)
   b. \( 2 \sin 4.8 \cos 4.8 \)
   c. \( \cos^2 0.8 - \sin^2 0.8 \)
   d. \( \cos 0.6 \cos 2.1 + \sin 0.6 \sin 2.1 \)

4. Use a sum or difference identity to find the exact value of each expression.
   a. \( \sin\left(-\frac{\pi}{12}\right) \)
   b. \( \sin 105^\circ \)
   c. \( \cos 285^\circ \)

5. Find the exact values of \( \sin 2x \), \( \cos 2x \), and \( \tan 2x \) for each set of conditions.
   a. \( \sin x = \frac{3}{5}, 0 \leq x \leq \frac{\pi}{2} \)
   b. \( \cos x = -\frac{5}{13}, \frac{\pi}{2} \leq x \leq \pi \)
LESSON 0.1  •  Pictures, Graphs, and Diagrams

1. a. 2  
   b. \( \frac{3}{2} \)  
   c. \(-\frac{2}{5}\)
2. a. \(a = 12\)  
   b. \(b = 36\)  
   c. \(c = 33\)  
   d. \(d = 78\)  
   e. \(w = 9\)  
   f. \(x = 280\)
3. a. \(\frac{7}{3}\)  
   b. \(-\frac{5}{2}\)  
   c. 1
4. a. \(\frac{7}{8}\)  
   b. \(\frac{13}{10}\)  
   c. \(\frac{5}{9}\)
5.  

LESSON 0.2  •  Symbolic Representation

1. a. Subtract 7 from both sides.  
   b. Divide both sides by 8.  
   c. Multiply both sides by \(-11\).
2. a. \(a = 27\)  
   b. \(b = 9\)  
   c. \(c = 15\)  
   d. \(d = -4\)  
   e. \(p = -\frac{7}{12}\), or \(-0.583\)  
   f. \(q = \frac{3}{4}\), or 0.75
3. a. \(-21 + 3y\), or \(3y - 21\)  
   b. \(-144q + 12q^2\), or \(12q^2 - 144q\)  
   c. \(-7y^3 + 21y\)  
   d. \(6r^2 - 15r\)  
   e. \(48s^2 - 30s\)  
   f. \(16z^3 - 120z\)
4. a. \(-25\)  
   b. 36  
   c. 26.4  
   d. 34

LESSON 0.3  •  Organizing Information

1. a. \(2.6w - 10.4\)  
   b. \(-4.6 - 2y\), or \(-2y - 4.6\)  
   c. \(6.8s - 2.8t + 8.4\)  
   d. \(3u - 26\)  
   e. \(-8z + 12\), or \(12 - 8z\)
2. a. \(p = 3.5\), or \(p = \frac{7}{2}\)  
   b. \(q = -13.3\), or \(-\frac{133}{10}\)  
   c. \(s = -8\)  
   d. \(z = -13\)
3. a. \(m^{12}\)  
   b. \(p^{-10}\), or \(\frac{1}{p^{10}}\)  
   c. \(81a^8b^4\)  
   d. \(-10x\)
4. a. \(x^2 + 9x + 20\)  
   b. \(4m^2 - 12m + 9\)  
   c. \(49p^3 - 81\)
5. a. \(k = 0.25\); \(y = 0.25x\)  
   b. 30 problems

LESSON 1.1  •  Recursively Defined Sequences

1. a. \(d = -0.5\)  
   b. \(r = 2\)
2. a. \(-18, -12, -6, 0, 6, 12\); arithmetic
   b. 0.5, 1.5, 4.5, 13.5, 40.5, 121.5; geometric
3. a. \(u_1 = 17.25\) and \(u_n = u_{n-1} - 2.31\) where \(n \geq 2\)
   b. \(u_1 = -2\) and \(u_n = -2u_{n-1}\) where \(n \geq 2\)
   c. \(u_1 = -15.09\)  
   d. \(u_1 = -32,768\)
4. a. Arithmetic; \(d = 50\)  
   b. Geometric; \(r = 1.05\)
5. \(u_1 = 12\) and \(u_n = u_{n-1} - 4\) where \(n \geq 2\)
   \(u_{42} = -152\)

LESSON 1.2  •  Modeling Growth and Decay

1. a. 3; growth; 200% increase
   b. 0.2; decay; 80% decrease
   c. 1.1; growth; 10% increase
   d. 0.6; decay; 40% decrease
2. a. \(u_0 = 42\) and \(u_n = 3u_{n-1}\) where \(n \geq 1\)
   b. \(u_0 = 19.2\) and \(u_n = 0.2u_{n-1}\) where \(n \geq 1\)
   c. \(u_0 = 90\) and \(u_n = 1.1u_{n-1}\) where \(n \geq 1\)
   d. \(u_0 = 144.9459\)
   e. \(u_0 = 1800\) and \(u_n = 0.6u_{n-1}\) where \(n \geq 1\)
   f. \(u_0 = 139.968\)
3. a. \((1 - 0.19)y\), or \(0.81y\)
   b. \((2 - 0.33)A\), or \(1.67A\)
   c. \((1 - 0.72)u_{n-1}\), or \(0.28u_{n-1}\)
   d. \((3 - 0.5)u_{n-1}\), or \(2.5u_{n-1}\)
4. a. \(u_0 = 3\) and \(u_n = -3u_{n-1} + 0.5\) where \(n \geq 1\), or \(u_1 = 3\) and \(u_n = -3u_{n-1} + 0.5\) where \(n \geq 2\)
5. a. C  
   b. A  
   c. B

LESSON 1.3  •  A First Look at Limits

1. a. \(u_1 = 33\), \(u_2 = 41\), \(u_3 = 49\); arithmetic; increasing
   b. \(u_1 = 1\), \(u_2 = 0.1\), \(u_3 = 0.01\); geometric; decreasing
   c. \(u_1 = 41.1\), \(u_2 = 34.2\), \(u_3 = 27.3\); arithmetic; decreasing
   d. \(u_1 = 125\), \(u_2 = 50\), \(u_3 = 35\); shifted geometric; decreasing
2. a. \( r = 300 \)  b. \( s = 100 \)  c. \( t = 0 \)
   d. No solution  e. \( w = -50 \)  f. \( z = 56 \)

3. a. 100  b. 20  c. 0  d. 100

4. a. \( u_0 = 0 \) and \( u_n = 0.8u_{n-1} + 20 \) where \( n \geq 1 \)
   b. \( u_0 = 100 \) and \( u_n = 1.1u_{n-1} + 50 \) where \( n \geq 1 \)
   c. \( u_0 = 50 \) and \( u_n = 0.6u_{n-1} + 6 \) where \( n \geq 1 \)
   d. \( u_0 = 40 \) and \( u_n = 1.6u_{n-1} - 20 \) where \( n \geq 1 \)

**LESSON 1.4 • Graphing Sequences**

1. Sample answers:
   a. \( (0, 2), (1, 10), (2, 18), (3, 26), (4, 34) \)
   b. \( (0, 10), (1, 1), (2, 0.1), (3, 0.01), (4, 0.001) \)
   c. \( (0, 0), (1, 10), (2, 35), (3, 97.5), (253.75) \)
   d. \( (0, 150), (1, 110), (2, 78), (3, 52.4), (4, 31.92) \)

2. a. C; geometric  b. A; arithmetic  c. B; geometric

3. a. Arithmetic, linear, decreasing
   b. Shifted geometric, nonlinear, decreasing
   c. Geometric, nonlinear, increasing
   d. Geometric, nonlinear, decreasing

**LESSON 1.5 • Loans and Investments**

1. a. Investment; principal: $1,000; deposit: $100
   b. Loan; principal: $130,000; deposit: $1,055
   c. Investment; principal: $1,825; deposit: $120

2. a. 4%; annually
   b. 6.25%, or 6\( \frac{1}{2} \)%; quarterly
   c. 7.5%, or 7\( \frac{3}{4} \)%; monthly

3. a. $100.00  b. $6.33

4. a. \( u_0 = 144,500 \) and \( u_n = (1 + 0.062)u_{n-1} - 990 \)
   where \( n \geq 1 \)
   b. \( u_0 = 0 \) and \( u_n = (1 + 0.0375)u_{n-1} + 225 \)
   where \( n \geq 1 \)

**LESSON 2.1 • Box Plots**

1. a. Mean: 6.3; median: 7; mode: 7
   b. Mean: 182; median: 180; mode: none
   c. Mean: 27.7; median: 32; mode: none
   d. Mean: 8; median: 8.8; modes: 5.3, 9.2

2. a. 500  b. 250  c. 500
   d. 250  e. 750  f. 750

3. a. 2, 3, 6, 9, 10  b. 0, 30, 50, 80, 95
   c. 1, 2.5, 4, 8, 9  d. 16, 36, 52, 60, 70
   e. 16.7, 18.65, 20.95, 29.5, 33.9

4. a. Median = 18; range = 11; IQR = 8
   b. Median = 449.5; range = 766; IQR = 568
   c. B  d. C

**LESSON 2.2 • Measures of Spread**

1. a. \( \bar{x} = 18 \); deviations: \(-5.6, 8.3, -8.2, 15.9, -10.4\)
   \( s = 11.5 \)
   b. \( \bar{x} = 421 \); deviations: \(-186, -8, 84, -310, 279, -205, -64\)
   \( s = 208.9 \)

2. a. \( \bar{w} = 12.5 \) cm; \( s = 4.80 \) cm
   b. \( \bar{w} = 13.80 \) cm; \( s = 2.88 \) cm
   c. \( \bar{w} = 5.6, 8.3, 10.4, 15.9, 16.7, 18.65, 20.95, 29.5, 33.9 \)

3. Sample answer: \{16, 9, 13, 9\}

4. a. 105  b. None  c. 5, 95

**LESSON 2.3 • Histograms and Percentile Ranks**

1. a. 25%
   b. Ages of Family Members

2. a. Bin width: 8; 29 values; bin 16–24
   b. Bin width: 12; 72 values; bin 24–36
   c. Bin width: 35; 1168 values; bin 140–175

3. a. 54th percentile  b. 62nd percentile

**LESSON 3.1 • Linear Equations and Arithmetic Sequences**

1. a. \( u_n = 18.25 - 4.75n \)  b. \( t_n = 100n \)

2. a. \( u_1 = 35 \) and \( u_n = u_{n-1} - 7 \) where \( n \geq 2 \);
   common difference: \(-7\); \( u_0 = 42 \)
   b. Slope: \(-7\); \( y\)-intercept: 42
   c. \( y = -7x + 42 \)

3. a. 3  b. \(-1\)  c. 0.6

4. a. \( y = 11 + 9x \)  b. \( y = -7.5 - 12.5x \)

**LESSON 3.2 • Revisiting Slope**

1. a. 3  b. \(-\frac{7}{5}\)  c. \(-\frac{5}{3}\)

2. a. -2.5  b. -4  c. -0.3

3. a. \( y = 14 \)  b. \( a = 390 \)

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4. a. \( y = 4 + \frac{4}{3}x \); \( y = -4 - \frac{4}{3}x \)
   b. \( y = 1 - 2x \); \( y = 1 + \frac{1}{2}x \)
   c. The constants and \( x \)-coefficients are negatives of each other. The lines share the same \( x \)-intercept.
   d. The equations have the same constant, and the \( x \)-coefficients are negative reciprocals of each other. The lines share the same \( y \)-intercept and are perpendicular.

LESSON 3.3 • Fitting a Line to Data

1. a. \( y = 3 + 2(x - 1) \)   b. \( y = 2 - \frac{3}{2}(x + 3) \)
2. a. \( y = 10 + 0.75(x + 4) \)   b. \( y = -5 - 4(x - 2) \)
3. a. \( d = -91 \)   b. \( x = 18 \)   c. \( n = 28 \)
4. a. Sample answer: The points go up to the right. My line has 5 points above the line and 5 points below. They are not concentrated at either end.
   b. Sample answer: The points go down to the right. My line has 5 points above the line, 5 points below, and 2 points on the line. They are not concentrated, though the 2 points on the line are on the right side.

LESSON 3.4 • The Median-Median Line

1. a. 11-11-11   b. 21-22-21   c. 19-19-19
2. a. (8, 8.5)   b. (3, 4)
   c. (15.5, 18.5)   d. (3.3, 4.4)
3. a. \( y = 8 - 2(x - 5) \), or \( y = 2 - 2(x - 8) \)
   b. \( y = 6 - 1(x + 1) \), or \( y = -4 - 1(x - 9) \)
   c. \( y = -14 + 0.6(x - 20) \), or \( y = 16 + 0.6(x + 30) \)
   d. \( y = -22.8 - 3(x - 4.4) \), or \( y = 34.2 - 3(x - 25.2) \)
4. a. \( y = 2x + 9 \)   b. \( y = 0.8x + 24.4 \)

LESSON 3.5 • Prediction and Accuracy

1. a. Above   b. Below
2. 4, -2, 9, 32

3. Sample answers are calculated using 1910–2000 as enrollment years.
   a. \( \hat{y} \approx 286.8x - 529,431.2 \); no, there cannot be a negative number of students enrolled.
   b. \(-542.8; 353.2; 1585.2; -1527.0; -4718.0; 2485.2; 9985.2; 3218.2; -757.8; 2643.2 \)
   c. 4349.1 students
   d. In general, the enrollment predicted by the median-median line will be within about 4349 students of the actual value.
   e. 2013–14 enrollment: 48,184 students; residual: 1,553
4. 2.93

LESSON 3.6 • Linear Systems

1. a. (0, 2)   b. (–2, 4)   c. (5, 6)
2. Sample answers:
   a. \( \begin{align*}
   4x + 5y &= 40 \\
   4x - 5y &= 0
   \end{align*} \)
   b. \( \begin{align*}
   5x - 2y &= -31 \\
   2x + 3y &= 18
   \end{align*} \)
   c. \( \begin{align*}
   x + 2y &= 24 \\
   x - 2y &= -18
   \end{align*} \)
3. a. \( y = -4 - 0.5(x - 5) \)   b. \( y = 18 - 4x \)
4. a. \( x = 1 \)   b. \( t = 36.6 \)
5. a. (–0.5, 4)   b. \( \left( \frac{40}{9}, 0 \right) \)   c. (3, 9)

LESSON 3.7 • Substitution and Elimination

1. a. \( s = r - 20 \)
   b. \( x = \frac{8y + 10}{5} \), or \( x = \frac{8y}{2} + 2 \)
   c. \( n = \frac{0.2m - \frac{1}{2}}{0.5} \), or \( n = 0.4m - 2 \)
   d. \( y = -\frac{-250x - 50}{400} \), or \( y = -0.625x - 0.125 \)
2. a. \( \left( \frac{3}{4}, \frac{1}{4} \right) \)
   b. (3.6, 2.4)
LESSON 4.1 • Interpreting Graphs

1. a. Increasing  
   b. Increasing and then decreasing
2. a. The child's height at birth  
   b. The height of the top of the building
3. Possible answer:

   ![Graph](image)

4. a. Independent variable: time; dependent variable: temperature. Sample graph:

   ![Graph](image)

   b. Independent variable: time; dependent variable: speed. Sample graph:

   ![Graph](image)

LESSON 4.2 • Function Notation

1. a. Function. Each x-value has only one y-value. Also, no vertical line crosses the graph more than once.
   b. Not a function. There are x-values that are paired with two y-values. Also, vertical lines can be drawn that cross the graph more than once.
2. a. \( f\left(-\frac{1}{4}\right) = 0, f(0) = -1, f(0.75) = -2 \)
   \( f(2) = -3, f(12) = -7 \)
   b. \( f(-4) = \frac{-1}{4}, f(0) = -\frac{1}{2}, f(5) = 2 \)
   \( f(8) = \frac{1}{2}, f(24) = \frac{1}{10} \)
3. a. -10  
   b. 135  
   c. 0.5  
   d. 13
4. a. If \( t \) is the time driven and \( d \) is the distance driven, then \( d = 65t \).
   b. If \( m \) is the miles driven, \( d \) is the number of days the van is rented, and \( c \) is the cost of renting the van, then \( c = 45d + 0.22m \).

LESSON 4.3 • Lines in Motion

1. a. Down 3 units  
   b. Left 6 units  
   c. Right 7 units and up 5 units
2. a. \(-4(x - 2) = -4x + 8\)
   b. \(3 + 2(x + 4) = 2x + 11\)
   c. \(2(x - 5) + 1 = 2x - 9\)
   d. \(3 + 8 - (x + 6) = -x + 5\)
3. a. \( y = -1.2(x - 3), \) or \( y = -1.2x + 3.6 \)
   b. \( y = 5 - (x + 2), \) or \( y = 3 - x \)
   c. \( y = -1 + \frac{1}{2}(x - 4), \) or \( y = -3 + \frac{1}{2}x \)
4. a. \( y = -2 + f(x) \)
   b. \( y = 3 + f(x) \)
   c. \( y = f(x - 3) \)
   d. \( y = -3 + f(x + 2) \)

LESSON 4.4 • Translations and the Quadratic Family

1. a. Vertically -6 units  
   b. Horizontally -5 units  
   c. Horizontally 3 units and vertically -9 units
2. a. (0, 3)  
   b. (2, 0)  
   c. (-5, 8)
3. a. \( y = (x + 3)^2 \)
   b. \( y = x^2 + 1 \)
4. a. Translated vertically −1 unit
   b. Translated horizontally 5 units
5. a. \(x = 5\) or \(x = −5\)  
   b. \(x = 8\) or \(x = −8\)  
   c. \(x = 13\) or \(x = −7\)  
   d. \(x = 5\) or \(x = −19\)  
   e. \(x = 2\) or \(x = −10\)  
   f. \(x = −5 \pm \sqrt{23}\)

LESSON 4.5 • Reflections and the Square Root Family

1. a. Translated horizontally −6 units  
   b. Translated vertically 5 units  
   c. Translated vertically −1 unit  
   d. Translated horizontally 8 units
2. a. \(y = \sqrt{x + 3}\)  
   b. \(y = −x^2 + 4\)  
   c. \(y = \sqrt{x − 4}\)  
   d. \(y = −(x + 2)^2\)  
   e. \(y = \sqrt{−x + 2}\)  
   f. \(y = −(x − 2)^2 + 3\)
3. a.  
   b. 
   c. 

LESSON 4.6 • Dilations and the Absolute-Value Family

1. a. \(y = \sqrt{x + 3} − 4\)  
   b. \(y = \frac{−x + 3}{2} + 1\)  
   c. \(y = |x + 3| + 2\)  
   d. \(y = −(x − 5)^2 + 4\)  
2. a. Horizontal translation 3 units  
   b. Vertical dilation by a factor of 1.5 and horizontal dilation by a factor of 2  
   c. Vertical dilation by a factor of 2; horizontal translation 1 unit and vertical translation −4 units

LESSON 4.7 • Transformations and the Circle Family

1. a. \(x^2 + y^2 = 4\)  
   b. \(x^2 + y^2 = 25\)  
   c. \((x − 2)^2 + y^2 = 1\)  
   d. \(x^2 + (y + 3)^2 = 9\)  
   e. \((x + 4)^2 + (y + 1)^2 = 4\)
2. a. \(-f(x) = −\sqrt{1 − x^2}\)  
   b. \(f(−x) = \sqrt{1 − x^2}\)  
   c. \(2f(x) = 2\sqrt{1 − x^2}\)  
   d. \(f(2x) = \sqrt{1 − (2x)^2} = \sqrt{1 − 4x^2}\)
3. a. \(x\)-intercepts: −1, 1; \(y\)-intercept: −1  
   b. \(x\)-intercepts: −1, 1; \(y\)-intercept: −2  
   c. \(x\)-intercepts: −0.5, 0.5; \(y\)-intercept: 1  
   d. \(x\)-intercepts: −0.25, 0.25; \(y\)-intercept: −2  
   e. \(x\)-intercepts: −3, 3; \(y\)-intercept: −1  
   f. \(x\)-intercepts: −4, 4; \(y\)-intercept: 2
4. a. \(\left(\frac{x}{2}\right)^2 + \left(\frac{y}{2}\right)^2 = 1\);  
   b. \(\left(\frac{x}{4}\right)^2 + \left(\frac{y}{3}\right)^2 = 1\);  
   circle  
   ellipse
LESSON 4.8 · Compositions of Functions

1. a. \{-3, -2, -1, 1, 3, 5\}
   b. \{-2, -1, 0\}
   c. 1
   d. -1
   e. 6
   f. 0
2. a. \((2x - 2)^2 + 1 = 4x^2 - 8x + 5\)
   b. 260
   c. 85
   d. \(a^4 + 4a^2 + 4\)
3. a. \(f(x) = 1.06x\)
   b. \(g(x) = 0.15x\)
   c. \(gf(x) = 0.15(1.06x) = 0.159x\)
   d. Marla’s way: $7.95; Shamim’s way: $7.50

LESSON 5.1 · Exponential Functions

1. a. \(r(8) \approx 427.9629\)
   b. \(j(10) \approx 21.9282\)
2. a. \(a_1 = 9.6, a_2 = 7.68, a_3 = 6.144; y = 12(0.8)^x\)
   b. \(u_1 = 106.05, u_2 = 222.705, u_3 = 467.6805; y = 50.5(2.1)^x\)
3. a. \(f(0) = 2000, f(1) = 1800, f(2) = 1620; u_0 = 2000, u_n = 0.9u_{n-1}\) where \(n \geq 1\); exponential decay
   b. \(f(0) = 3000, f(1) = 3003, f(2) = 3006.003; u_0 = 3000, u_n = 1.001u_{n-1}\) where \(n \geq 1\); exponential growth
   c. \(f(0) = 0.1, f(1) = 0.05, f(2) = 0.025; u_0 = 0.1, u_n = 0.5u_{n-1}\) where \(n \geq 1\); exponential decay
4. a. 0.75; decrease of 25%
   b. 0.8; decrease of 11.1%
   c. 1.6; increase of 60%
5. a. \(u_0 = 17500, u_n = 0.84u_{n-1}\) where \(n \geq 1\)
   b. Let \(x\) represent the number of years after the car was purchased and \(y\) represent the value of the car. \(y = 17500(0.84)^x\)

LESSON 5.2 · Properties of Exponents and Power Functions

1. a. \(\frac{1}{9}\)
   b. \(\frac{1}{343}\)
   c. \(-\frac{1}{256}\)
   d. \(-\frac{1}{125}\)
   e. \(-\frac{25}{9}\)
   f. \(\frac{36}{25}\)
2. a. \(36x^8\)
   b. \(-120x^{-20}\)
   c. \(x^{18}\)
   d. \(11x^7\)
   e. \(125x^{15}\)
   f. \(-\frac{1}{125}x^{18}, or -0.008x^{18}\)

LESSON 5.3 · Rational Exponents and Roots

3. a. \(x = -5\)
   b. \(x = \frac{2}{3}\)
   c. \(x = -2\)
4. a. \(x = 5.62\)
   b. \(x = 0.39\)
   c. \(x \approx 578703.70\)
   d. \(x \approx 0.91\)
   e. \(x = 1.5\)
   f. \(x \approx 0.79\)

LESSON 5.4 · Applications of Exponential and Power Equations

1. a. \(x = 17.58\)
   b. \(x = 625\)
   c. \(x \approx 1.46\)
   d. \(x = 65.80\)
   e. \(x = 0.05\)
   f. \(x \approx 0.03\)
2. a. \(4x^6\)
   b. \(27x^9\)
   c. \(7x^{-5}\)
   d. \(81x^{-12}\)
   e. \(1000x^6\)
   f. \(-5x^{-5}\)
3. a. 3.5%
   b. 3.9%
4. a. Let \(t\) represent the year and \(P\) represent the population. \(P = 23,000(0.96)^{t-2002}\)
   b. 16,592
   c. 2018

LESSON 5.5 · Building Inverses of Functions

1. a. \(f: (-2, -10), (0, -4), \left(\frac{4}{3}, 0\right), (4, 8)\)
   \(f^{-1}: (-10, -2), (-4, 0), (0, \frac{4}{3}), (8, 4)\)
   b. \(f: (-3, 29), (-1, 3), (2, 6), (5, 123)\)
   \(f^{-1}: (-29, -3), (-3, 1), (6, 2), (123, 5)\)
2. a. Function;
   b. Not a function;
   \(y = \frac{1}{2}x + \frac{5}{2}\)
   \(x = |y|\)

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c. Not a function; 
\[ y = \pm \sqrt{x} + 4 \]

3. a. $61$
   b. \[ p^{-1}(x) = \frac{1}{8}\left(\sqrt{\left(x - \frac{25}{4}\right)^3} - 3\right) \]
   c. 15\(\frac{3}{4}\) person-hours (or 15 person-hours if fractional person-hours are not allowed)

**LESSON 5.6 - Logarithmic Functions**

1. a. \[ 3^x = \frac{1}{81}; \quad x = -4 \]
   b. \[ x^{1/4} = \sqrt[4]{12}; \quad x = 12 \]
   c. \[ 4^x = 32; \quad x = 2.5 \]
   d. \[ 10^1 = x; \quad x = 10 \]
   e. \[ x^3 = 125; \quad x = 5 \]
   f. \[ 20^1 = x; \quad x = 20 \]
2. a. 4 
   b. \(\frac{1}{2}\) 
   c. -1 
   d. -5 
   e. \(\frac{2}{3}\) 
   f. 9 
3. a. \[ y = 10^x - 3 \]
   b. \[ y = \log(x - 1) + 3 \]
   c. \[ y = \frac{1}{2}(10^{2x+2}) \]
4. a. 2.9746 
   b. -0.0959 
   c. 3.3147 
   d. 3.6789 
   e. 0.8146 
   f. 0.7039

**LESSON 5.7 - Properties of Logarithms**

1. a. \(\log 3\) 
   b. \(\log \frac{1}{16}\) 
   c. \(\log 25\) 
2. a. \(\log_a a + \frac{1}{2}\log b - 4 \log c\) 
   b. \(\frac{1}{2}\log x + \frac{1}{3}\log y + \frac{3}{4}\log t\)
   c. \(\frac{1}{3}\log a + \frac{1}{2}\log b + \frac{1}{3}\log c - \frac{1}{4}\log x\)
3. a. False 
   b. True 
   c. False 
   d. True 
   e. True 
   f. True 
   g. False 
   h. True 
4. a. \(\log a^b; \) definition of negative exponents 
   b. \(a^{-b}; \) definition of negative exponents 
   c. \(q^a \cdot q^b; \) product property of exponents 
   d. \(m \log a; \) power property of logarithms 
   e. \(e^{m \log a}; \) power of a product property 
   f. \(\log_a x + \log_b y; \) product property of logarithms 
   g. \(e^{\log a}; \) power of a quotient property 
   h. \(\sqrt{e}; \) definition of rational exponents 
   i. \(\log_a x; \) change-of-base property

**LESSON 5.8 - Applications of Logarithms**

1. a. \(x = 9\) 
   b. \(x \approx 1.5111\) 
   c. \(x \approx 11.1144\) 
2. a. 14.2 yr 
   b. 19.3 yr 
3. a. 25,118,864I_0 
   b. 1,995,262I_0 
   c. The intensity of the 1999 earthquake was about 12.6 times as great as that of the 1998 earthquake.
4. 6 years (72 months)

**LESSON 6.1 - Matrix Representations**

1. a. 
   b. 
   c. 327

**LESSON 6.2 - Matrix Operations**

1. a. 
   b. 
   c. 
   d. 
   e. 
   f. 
   g. 
   h. 
2. a. 
   b. 
   c. 

- Impossible because the inside dimensions aren’t the same.
3. a. 
   b. 
   c. 

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LESSON 6.3 • Solving Systems with Inverse Matrices

1. a. \[
\begin{align*}
4x - 3y &= 5 \\
2x + 6y &= -1
\end{align*}
\]
   b. \[
\begin{align*}
0.5x + 0.8y &= 4 \\
0.1x - 0.2y &= 1.5
\end{align*}
\]
2. \[
\begin{bmatrix}
2 & 3 & -1 \\
3 & 0 & 4 \\
0 & 1 & -2
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix}
= 
\begin{bmatrix}
-1 \\
-5 \\
-3
\end{bmatrix}
\]
3. a. Not inverses  
   b. Inverses
4. a. \[
\begin{bmatrix}
2 & 2.5 \\
1 & 1.5
\end{bmatrix}
\]
   b. Inverse does not exist.
   c. \[
\begin{bmatrix}
0.2 & 0.4 & 0.2 \\
-0.4 & 0.2 & -0.4 \\
-0.8 & 0.4 & 0.2
\end{bmatrix}
\]
5. a. \[
\begin{align*}
4c + 5d &= 146 \\
6c + 3d &= 138
\end{align*}
\]
   b. \[
\begin{bmatrix}
\frac{-1}{6} & \frac{5}{18} \\
\frac{1}{3} & \frac{-2}{9}
\end{bmatrix}
\begin{bmatrix}
146 \\
138
\end{bmatrix}
\]
   c. \[
\begin{bmatrix}
14 \\
18
\end{bmatrix}
\]
   CD: $14; DVD: $18

LESSON 6.4 • Row Reduction Method

1. a. \[
\begin{align*}
3x - y &= 5 \\
x + 4y &= -2
\end{align*}
\]
   b. \[
\begin{align*}
4x - y + z &= 6 \\
-2x + 3y - 2z &= 5 \\
3x + 4y - 4z &= 0
\end{align*}
\]
2. \[
\begin{bmatrix}
1 & 1 & -1 & 0 \\
2 & 3 & -3 & -3 \\
-1 & -2 & 2 & 3
\end{bmatrix}
\begin{bmatrix}
0 \\
1 \\
-4
\end{bmatrix}
\]
3. \[
\begin{bmatrix}
4 & -2 & 5 \\
0 & 1 & -4
\end{bmatrix}
\]
   b. \[
\begin{bmatrix}
1 & 3 & 5 & -2 \\
0 & -13 & -13 & 6 \\
-2 & 4 & 6 & 1
\end{bmatrix}
\]
4. \[
\begin{bmatrix}
m + b + g &= 475 \\
35m + 25b + 15g &= 13275 \\
m - b &= 45
\end{bmatrix}
\]
   a. \[
\begin{bmatrix}
1 & 1 & 1 & 475 \\
35 & 25 & 15 & 13275 \\
1 & -1 & 0 & 45
\end{bmatrix}
\]
LESSON 6.5 • Systems of Inequalities

1. a. \[y \geq -4 + 4 \frac{5}{3} \\
y \leq - \frac{9}{8} + \frac{1}{4} \]
2. a.  
   b.  
   c.  
   d.  

LESSON 6.6 • Linear Programming

1. 25x + 14y; maximized
2. a. 3.5x + 5y ≤ 21  
   b. 3x + 4y ≤ 14  
   c.  

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d. Vertices: \((\frac{14}{3}, 0), (6, 0), (0, 4.2), (0, 3.5)\)

\[
\begin{array}{c|c|c|c|c}
\hline
n & 1 & 2 & 3 & 4 \\
\hline
\text{nth triangular number} & 1 & 3 & 6 & 10 \\
\hline
\text{D_1} = \{2, 3, 4, 5, 6, 7\} & & & & \\
\text{D_2} = \{1, 1, 1, 1, 1\} & & & & \\
\text{c. 2} & & & & \\
\text{d. } t(n) = \frac{1}{2}n^2 + \frac{1}{2}n, \text{ or } t(n) = 0.5n^2 + 0.5n & & & & \\
\hline
\end{array}
\]

**LESSON 7.2 • Equivalent Quadratic Forms**

1. a. General form  
   b. Vertex form  
   c. Factored form

2. a. \(y = x^2 - 6x + 9\)  
   b. \(y = -5x^2 - 5x\)  
   c. \(y = 3x^2 - 9x - 3.25\)

3. a. \((0, 0)\)  
   b. \((1, 6)\)  
   c. \((-4, 6.5)\)

4. a. \(x = 1\) and \(x = -6\)  
   b. \(x = 0\) and \(x = 5\)  
   c. \(x = 7.5\)

5. a. \(x = 1.5\)  
   b. \((-1.5, -5)\); minimum  
   c. \(y = 2(x + 1.5)^2 - 5\)

**LESSON 7.3 • Completing the Square**

1. a. \((x + 5)^2\)  
   b. \((x - \frac{1}{2})^2\)  
   c. \((3x - 4)^2\)

2. a. 81  
   b. \(25\) or 6.25  
   c. 4.6225

3. a. \(y = (x + 7)^2 + 1\)  
   b. \(y = 5(x - 1)^2 - 8\)  
   c. \(y = 2(x + 1.25)^2 - 3.125\)

4. a. \((-2, -16)\); minimum  
   b. \((-1, 3.5)\); minimum  
   c. \((-4.5, -30.25)\); minimum

5. a. \(3x^2 + 6x + 2; a = 3, b = 6, c = 2\)  
   b. \(-2x^2 + 16x; a = -2, b = 16, c = 0\)  
   c. \(2x^2 + 7x - 15; a = 2, b = 7, c = -15\)

6. a. \(h = -4.9t^2 + 14.7t + 75\)  
   b. 1.5 s; 86.025 m  
   c. 5.69 s

**LESSON 7.4 • The Quadratic Formula**

1. a. 0.742  
   b. 0.293  
   c. 1.693  
   d. 2.436

2. a. \(x = -5\) or \(x = 2\)  
   b. \(x = -\frac{3}{2}\) or \(x = 4\)  
   c. \(x = \pm \frac{7}{5}\)  
   d. \(d = \frac{-7 \pm \sqrt{65}}{8}\)

3. a. \(y = (x + 8)(x - 3)\)  
   b. \(y = 2(x - 3)(x - 1)\)  
   c. \(y = 4(x + 1)(x - 0.5)\)

4. a. \(y = -x^2 - 6x - 8\)  
   b. \(y = -x^2 + 13x\)  
   c. \(y = -0.25x^2 + 2.4x - 5.76\)

**LESSON 7.5 • Complex Numbers**

1. a. \(-6 + 7i\)  
   b. \(3.5 - 3.8i\)  
   c. \(4 + 24i\)  
   d. \(9.4 + 3.6i\)

2. a. \(5 + 4i\)  
   b. \(-7i\)  
   c. \(-3.25 - 4.82i\)

3. a. \(\frac{3}{5} - \frac{1}{3}i\) or \(0.6 - 0.2i\)  
   b. \(i\), or \(0 + i\)  
   c. \(\frac{5}{6} - \frac{1}{2}i\)

4. a. \(x = 1 \pm 2i\); complex  
   b. \(x = \pm i\sqrt{7};\) imaginary and complex  
   c. \(x = 5 \pm \frac{\sqrt{29}}{2};\) real and complex  
   d. \(x = \frac{-1 \pm i\sqrt{3}}{2};\) complex  
   e. \(x = \pm 3i;\) or \(x = \pm 1.5i;\) imaginary and complex  
   f. \(x = \frac{-6 \pm \sqrt{140}}{2},\) or \(x = -3 \pm \sqrt{35};\) 
   \(\) real and complex

5. a. \(x^2 - 3x - 28 = 0\)  
   b. \(x^2 + 121 = 0\)  
   c. \(x^2 + 4x + 13 = 0\)

6. A: \(3i\), or \(0 + 3i\)  
   B: \(-3i\)  
   C: \(-4 + i\)
LESSON 7.6 • Factoring Polynomials

1. a. x-intercept: 8; y-intercept: –64
    b. x-intercepts: –4, –2; y-intercept: 24
    c. x-intercepts: 0, 2, –6; y-intercept: 0
2. a. \( y = -2(x + 3)(x - 4) \)
    b. \( y = 0.5(x + 4)(x - 2) \)
3. a. \( y = -2x^2 + 12.5 \)
    b. \( y = -0.5x^3 - 3x^2 + 4.5x^2 \)
    c. \( y = -x^3 + 144x \)
4. a. \( (x - 7)^2 \)
    b. \( x(x - 1)(x - 2) \)
    c. \( (x + 13)(x - 13) \)
    d. \( (x + \sqrt{15})(x - \sqrt{15}) \)
    e. \( (x + 1)(x - 1)(x + 3)(x - 3) \)
    f. \( 3(x - 1)(x + 4)(x - 2) \)
5. Possible graphs:
   a. 
   b. 

LESSON 7.7 • Higher-Degree Polynomials

1. a. i. \( x = -3, x = 0, x = 2; \) ii. \( x = -2, x = 2 \)
    b. i. 0; ii. 4
    c. i. 3; ii. 4
    d. i. \( y = x(x + 3)(x - 2); \) ii. \( y = 0.25(x + 2)^2(x - 2)^2 \)
2. a. \( y = 2(x - 1)(x - 5), \) or \( y = 2x^2 - 12x + 10 \)
    b. \( y = -x^2(x - 2)^2, \) or \( y = -x^4 + 4x^3 - 4x^2 \)
3. a. \( y = 2(x + 3)(x - 5); \) degree 2
    b. \( y = -(x - 2i)(x + 2i)(x + 2)^2(x - 5), \)
    or \( y = -(x^2 + 4)(x + 2)^2(x - 5); \) degree 5

LESSON 7.8 • More About Finding Solutions

1. a. \( 3x^2 - 2x - 15 \)
    b. \( x^3 + 4x^3 + 3x + 12 \)
2. a. Dividend: \(-3x^4 - 5x^3 - 35x + 7; \) divisor: \( x + 3 \)
    b. 9 36 3
    c. –3 4 –12 1 4
    d. Quotient: \(-3x^3 + 4x^2 - 12x + 1; \) remainder: 4
3. a. \( 2x^2 - 9x + 2 = (x - 5)(2x + 1) + 7 \)
    b. \( 2x^2 - 5x^2 + 8x - 5 = (x - 1)(2x^2 - 3x + 5) \)
4. a. \( \pm 1, \pm 2, \pm 4, \pm 8 \)
    b. \( \pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 10, \pm 15, \pm 30, \pm 1 \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{5}{2}, \pm \frac{15}{2} \)
5. a. \( x = 5, x = 3i, \) and \( x = -3i; \)
    \( y = (x - 5)(x - 3i)(x + 3i), \)
    or \( y = (x - 5)(x^2 + 9) \)
    b. \( x = -2, x = -\frac{2}{3}, \) and \( x = \frac{1}{2}i; \)
    \( y = 6(x + 2)(x + \frac{3}{4})(x - \frac{1}{2}), \)
    or \( y = (x + 2)(3x + 4)(2x - 1) \)

LESSON 8.1 • Using the Distance Formula

1. a. 13 units
    b. 5 units
    c. \( \sqrt{20} \) units, or \( 2\sqrt{5} \) units
    d. \( \sqrt{80} \) units, or \( 4\sqrt{5} \) units
    e. \( \sqrt{25r^2 + 9} \) units
    f. \( \sqrt{5} \) units
2. a. \( y = -2 \) or \( y = 14 \)
    b. \( x = 4 \pm \sqrt{57} \)
3. a. \((x + 2)^2 + (y - 3)^2 = 25\)
    b. \(4x + 10y = 29\)
c. $x^2 - 6x + y^2 = 27$, or $y = \pm \sqrt{-x^2 + 6x + 27}$

\[ y = \pm \sqrt{-x^2 + 6x + 27} \]

**LESSON 8.2 • Circles and Ellipses**

1. a. $(0, 0); r = 4$  
   b. $(3, 0); r = 10$
   c. $(0.5, -0.5); r = 0.5$
   d. $(-1, 3); r = \frac{5}{2}$

2. a. $(0, 0)$; horizontal: 3; vertical: 5  
   b. $(2, 0)$; horizontal: 4; vertical: 2  
   c. $(-5, 4)$; horizontal: 3; vertical: 5

3. a. Major axis: $(0, 5), (0, -5)$; minor axis: $(3, 0), (-3, 0)$; foci: $(0, 4), (0, -4)$

\[ y = \pm \sqrt{-x^2 + 6x + 27} \]

b. Major axis: $(6, 0), (-2, 0)$; minor axis: $(2, 2), (2, -2)$; foci: $(2 + \sqrt{12}, 0), (2 - \sqrt{12}, 0)$, or $(2 + 2\sqrt{3}, 0), (2 - 2\sqrt{3}, 0)$

\[ y = \pm \sqrt{-x^2 + 6x + 27} \]

c. Major axis: $(-5, 9), (-5, -1)$; minor axis: $(-2, 4), (-8, 4)$; foci: $(-5, 8), (-5, 0)$

\[ y = \pm \sqrt{-x^2 + 6x + 27} \]

\[ \frac{x^2}{3^2} + \frac{y^2}{5^2} = 1 \]

**LESSON 8.3 • Parabolas**

1. a. $x = -4$  
   b. $(8.5, 0)$  
   c. $(0.5, -3)$

2. a. $(0, -5)$; upward; $x = 0$  
   b. $(0, 0)$; downward; $x = 0$  
   c. $(1, 0)$; right; $y = 0$  
   d. $(0, 3)$; left; $y = 3$  
   e. $(-1, -2)$; downward; $x = -1$  
   f. $(-5, 4)$; right; $y = 4$

\[ \frac{(x - 2)^2}{\frac{3}{2}} = y^2 \]

b. $(y - 3)^2 = x$

c. $\frac{y^2}{1} = x^2$

**LESSON 8.4 • Hyperbolas**

1. a. $(\frac{x^2}{3}) - (\frac{y^2}{2}) = 1$  
   b. $(\frac{y^2}{3}) - (\frac{x^2}{2}) = 1$

2. a. Asymptotes: $y = \pm x$; vertices: $(0, -1), (0, 1)$; foci: $(0, \sqrt{2}), (0, -\sqrt{2})$

\[ \frac{x^2}{3^2} - \frac{y^2}{2^2} = 1 \]

b. Asymptotes: $y = \frac{5}{3}x - \frac{5}{3}$ and $y = -\frac{5}{3}x + \frac{5}{3}$; vertices: $(-2, 0), (4, 0)$; foci: $(1 + \sqrt{34}, 0), (1 - \sqrt{34}, 0)$

\[ \frac{y^2}{5^2} - \frac{x^2}{3^2} = 1 \]

3. a. Hyperbola: $(\frac{y^2}{4}) - (\frac{x^2}{3}) = 1$  
   b. Ellipse: $(\frac{x^2}{5}) + (\frac{y^2}{3}) = 1$
LESSON 8.6 · Introduction to Rational Functions

1. a. \( f(x) = \frac{1}{x + 3} - 4 \) \hspace{1cm} b. \( f(x) = \frac{3}{x} \)

\[ \begin{align*}
\text{Graph 1:} & \quad y = \frac{1}{x + 3} - 4 \\
\text{Graph 2:} & \quad y = \frac{3}{x}
\end{align*} \]

2. a. Horizontal: \( y = 0 \); vertical: \( x = 0 \) \hspace{1cm} b. Horizontal: \( y = 0 \); vertical: \( x = 0 \) \hspace{1cm} c. Horizontal: \( y = 0 \); vertical: \( x = 0 \) \hspace{1cm} d. Horizontal: \( y = 5 \); vertical: \( x = 0 \) \hspace{1cm} e. Horizontal: \( y = -6 \); vertical: \( x = 0 \) \hspace{1cm} f. Horizontal: \( y = -1 \); vertical: \( x = 0 \)

3. a. \( x = 2 \) \hspace{1cm} b. \( x = -\frac{13}{2} \) \hspace{1cm} c. \( x = -3 \)

4. \( \frac{34 + x}{142 + x} = 0.280; \) 8 hits \hspace{1cm} \( \frac{78 + x}{120 + x} = 0.75; \) 48 mL

LESSON 8.7 · Graphs of Rational Functions

1. a. \( \frac{(x - 6)(x + 1)}{(x + 5)(x - 5)} \) \hspace{1cm} b. \( \frac{(x + 4)(x - 4)}{(2x - 3)(3x + 1)} \) \hspace{1cm} c. \( \frac{3(x + 1)(3x - 1)}{x(2x - 3)(x + 1)} \)

2. a. \( \frac{2 + 3x}{x} \) \hspace{1cm} b. \( \frac{6x + 13}{x + 5} \) \hspace{1cm} c. \( \frac{x - 19}{x + 3} \)

3. a. Vertical: \( x = 0 \); horizontal: \( y = 0 \) \hspace{1cm} b. Vertical: \( x = 2 \); horizontal: \( y = 0 \) \hspace{1cm} c. Vertical: \( x = -2, \) \( x = 2 \); horizontal: \( y = 0 \)

4. a. Vertical: \( x = 0 \); slant: \( y = x \) \hspace{1cm} b. Vertical: \( x = -2, \) \( x = 2 \); slant: \( y = x \) \hspace{1cm} c. Vertical: \( x = -2 \); slant: \( y = -x + 2 \)

5. a. \( (3, -1) \) \hspace{1cm} b. \( (-3, 2) \) \hspace{1cm} c. \( (-2, -7) \)

LESSON 8.8 · Operations with Rational Expressions

1. a. \( \frac{x(x - 4)}{(x + 3)(x - 4)} = \frac{x}{x + 3} \) \hspace{1cm} b. \( \frac{x + 7}{(x + 7)(x + 7)} = \frac{x - 7}{x + 7} \) \hspace{1cm} c. \( \frac{2x(x - 5)}{(3x + 4)(x - 5)} = \frac{2x}{3x + 4} \)

2. a. \( \frac{x + 4}{(x + 4)(x - 5)} \) \hspace{1cm} b. \( \frac{x + 4}{(x + 4)(x + 1)} \) \hspace{1cm} c. \( \frac{x - 2}{(x - 2)(x - 4), \) or \( (x - 2)^2(x - 4)} \) \hspace{1cm} d. \( x(x - 8)(x + 1)(2x + 1) \)

3. a. \( \frac{8x + 13}{(x + 2)(x - 1)(x + 1)} \) \hspace{1cm} b. \( \frac{-x^2 + 11x - 4}{(x + 7)(x - 7)(x - 1)} \) \hspace{1cm} c. \( 1 \) \hspace{1cm} d. \( 3x \)

4. a. \( \frac{1}{x(x + 2)} \) \hspace{1cm} b. \( \frac{1}{x + 3} \) \hspace{1cm} c. \( -\frac{1}{2} \)
**LESSON 9.1 • Arithmetic Series**

1. a. 5, 11, 17, 23, 29, 35; \( d = 6 \)
   
   b. 7.8, 5.5, 3.2, 0.9, -1.4, -3.7; \( d = -2.3 \)

2. a. \(-4 + (-3) + (-2) = -9\)

   b. \(-4 + (-1) + 2 + 5 = 2\)

   c. \(7 + 13 + 23 + 37 + 55 = 135\)

3. a. \(12\frac{3}{4}\) or 12.75  
   b. 105  
   c. 29,925

4. a. 17, 19, 21, 23, 25, 27

   b. \(u_1 = 17\) and \(u_n = u_{n-1} + 2\) where \(n \geq 2\)

   c. \(u_n = 17 + 2(n - 1)\), or \(u_n = 2n + 15\)

   d. 41 seats  
   e. 59 seats  
   f. 836 seats

**LESSON 9.2 • Infinite Geometric Series**

1. a. \(0.39 + 0.0039 + 0.000039 + \ldots\)

   b. \(u_1 = 0.39; r = 0.01\)  
   c. \(S = \frac{13}{33}\)

2. a. \(r = 0.8;\) convergent; \(S = 20\)

   b. \(r = -0.5;\) convergent; \(S = \frac{30}{3},\) or 6.7

   c. \(r = -1.1;\) not convergent

   d. \(r = 0.1;\) convergent; \(S = \frac{130}{9},\) or 14.4

3. a. \(-12\)  
   b. 2.2  
   c. 9.6

4. \(r = \frac{2}{3}\)

5. \(u_1 = 70.4; 70.4, -42.24, 25.344, -15.2064\)

6. About 227 cm

**LESSON 9.3 • Partial Sums of Geometric Series**

1. a. \(u_1 = 15, r = 0.6, n = 5\)

   b. \(u_1 = 30, r = 0.5, n = 8\)

2. a. \(0.049152\)

   b. \(u_2 = 0.768\)

   c. \(0.12288\)

   d. \(312.41802\)

3. a. \(u_1 = 5, d = 1.2, S_1_1 = 121\)

   b. \(u_1 = 150, r = -0.2, S_4 = 125.000064\)

   c. \(u_1 = 12.5, r = 1.1, S_1_5 = 397.1560212\)

   d. \(u_1 = 68.5, d = -3.5, S_5_0 = -862.5\)

4. a. 118,096  

   b. 5  

   c. 12  

   d. 0.5

5. a. Geometric; \(r = 1.05\)

   b. \$912  

   c. \$49,740

**LESSON 10.1 • Randomness and Probability**

1. a. \(0.495\)

   b. 0.316

   c. \(0.572\)

   d. Experimental

2. a. \(\frac{3}{18} \approx 0.167\)

   b. \(\frac{11}{18} \approx 0.611\)

   c. \(\frac{9}{18} \approx 0.5\)

   d. Theoretical

**LESSON 10.2 • Counting Outcomes and Tree Diagrams**

1. \(P(a) = 0.68, P(b) = 0.06, P(c) = 0.94, P(d) = 0.08, P(e) = 0.92, P(f) = 0.0408, P(g) = 0.2944\)

2. 
   
   ![Tree Diagram](image)

3. \(\frac{1}{12} = 0.083\)

   b. \(\frac{3}{12} = 0.25\)

3. \(650 = \frac{26}{105} \approx 0.248\)

4. a. 0.03  

   b. 0.45

**LESSON 10.3 • Mutually Exclusive Events and Venn Diagrams**

1. a. 0.21  

   b. 40

2. a. 0.79  

   b. 0.34  

   c. 0.66

3. a. 

   ![Venn Diagram](image)

   b. 0.13  

   c. 0.25

**LESSON 10.4 • Random Variables and Expected Value**

1. a. Random variable, discrete random variable

   b. Random variable

   c. Random variable, discrete random variable, geometric random variable

2. a. 0.35  

   b. 0.821

3. a. 

<table>
<thead>
<tr>
<th>(x_i)</th>
<th>10,000</th>
<th>1,000</th>
<th>100</th>
<th>20</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P(x_i))</td>
<td>0.00002</td>
<td>0.001</td>
<td>0.002</td>
<td>0.02</td>
<td>0.97698</td>
</tr>
</tbody>
</table>

   b. 48,849  

   c. \$1.80
LESSON 10.5 • Permutations and Probability

1. a. 720  b. 6,760,000  c. 60
d. 9,000,000  e. 112

2. a. 15  b. 420
c. \( n(n - 1)(n - 2) \), or \( n^2 + n \)
d. 90  e. 32,760
f. \( n(n - 1)(n - 2) \), or \( n^3 - 3n^2 + 2n \)

3. a. 720 ways  b. \( \frac{1}{720} \approx 0.001 \)  c. \( \frac{1}{6} \approx 0.167 \)

LESSON 10.6 • Combinations and Probability

1. a. 10  b. 56  c. 1
2. a. 20  b. 8  c. 20  d. 1
3. a. 20  b. 4,845  c. 1,848
4. a. 142,506  b. \( \frac{5}{30} \approx 0.167 \)
c. \( \frac{23,760}{142,506} \approx 0.167 \)

LESSON 10.7 • The Binomial Theorem and Pascal’s Triangle

1. a. \( 6xy^2 \)  b. \(-10m^2n^3 \)  c. \( 243b^5 \)
2. a. \( \frac{6}{64} = 0.09375 \)  b. \( \frac{42}{64} = 0.65625 \)
3. a. 0.668  b. 0.053  c. 0.993

4. What is the probability of at least 10 successes in 30 trials?

LESSON 11.1 • Experimental Design

1. a. Survey  b. Observational study  c. Survey
2. a. Sample answer: The sample is biased because the following segments of the population are underrepresented: people who are not home on Mondays and people who do not have their phone number listed in the phone book.
b. Sample answer: The sample is biased because the types of animals in the park probably do not represent the animal population overall. Also, the time of year or weather conditions other than temperature are not taken into consideration.
c. Sample answer: The sample is biased because it only includes citizens who are buying movie tickets.
3. No. The announcer is relying on information from only five people, which is likely not a large enough sample to represent all her listeners. Also, some listeners may be unwilling or unable to call in and therefore would not be represented in the statistic.
4. a. Experiment

   b. The treatment is eating fruit. It is assigned to half of his classmates, chosen randomly.

LESSON 11.2 • Probability Distributions

1. a. \( \frac{10}{18} \approx 0.556 \)  b. \( \frac{3}{18} \approx 0.167 \)
c. \( \frac{6}{18} \approx 0.333 \)  d. \( \frac{7}{18} \approx 0.389 \)

2. a. \( \frac{1}{12} \)  b. \( \frac{2}{12} \approx 0.167 \)  c. 5
   d. 3.9  e. 3.7

3. a. \( \frac{1}{8} \)  b. mean = median = 5
c. No. All x-values from 2 to 8 are equally likely and no other value has a greater probability, so there is no mode.

LESSON 11.3 • Normal Distributions

1. a. \( \mu = 32, \sigma = 2.5 \)  b. \( \mu = 100, \sigma \approx 15 \)
2. a. \( y = \frac{1}{2.5\sqrt{2\pi}} (\sqrt{e})^{-[(x-32)/2.5]^2} \)
b. \( y = \frac{1}{15\sqrt{2\pi}} (\sqrt{e})^{-[(x-100)/15]^2} \)
3. a. \( \mu = 55, \sigma = 8 \)  b. \( \mu = 4.8, \sigma = 0.75 \)
4. a. 0.4256  b. 0.6827
c. About 789  d. About 841

LESSON 11.4 • z-Values and Confidence Intervals

1. a. 69.4 in.  b. 66.1 in.  c. 57.3 in.
2. a. \( z = -1.5 \)  b. \( z = 2.2 \)  c. 0.819
3. a. (51.3, 53.9)  b. (50.5, 54.7)
c. (49.3, 55.9)  d. (48.8, 56.4)

LESSON 11.5 • Bivariate Data and Correlation

1. \[
\begin{array}{c|c|c|c|c|c}
  x & y & x - \bar{x} & y - \bar{y} & (x - \bar{x})(y - \bar{y}) \\
  \hline
  2 & 5 & -5 & -6.5 & 32.5 \\
  4 & 8 & -3 & -3.5 & 10.5 \\
  6 & 10 & -1 & -1.5 & 1.5 \\
  8 & 13 & 1 & 1.5 & 1.5 \\
  10 & 15 & 3 & 3.5 & 10.5 \\
  12 & 18 & 5 & 6.5 & 32.5 \\
\end{array}
\]

a. \( \bar{x} = 7, \bar{y} = 11.5 \)  b. 89
c. \( sx \approx 3.7417, sy \approx 4.7645 \)  d. \( r \approx 0.998 \)
e. There is a very strong positive correlation in the data.
2. a. Correlation. The people who take vitamins may take better care of their health, including having healthier diets, exercising more, and receiving better-quality health care.

b. Correlation. There may be a schedule conflict between the psychology class and the band class so that students cannot enroll in both.

LESSON 11.6 • The Least Squares Line

1. a. 1995    b. 14,102    c. 7.9057
d. 1119.4787  e. -0.9971

2. a. \( \hat{y} = 6.5 - 4.8(x - 3) \) or \( \hat{y} = 20.9 - 4.8x \)
b. \( \hat{y} = 8.3 + 0.3528(x - 28.4) \) or \( \hat{y} = -1.71952 + 0.3528x \)

3. a. 0    b. 28,570    c. 84.51

4. a. \( \hat{y} = 65.35 + 1.36x \)
b. The slope, 1.36, shows that, according to this model, the percentage of U.S. households with TV that have basic cable service has been increasing by about 1.36% per year. The \( y \)-intercept, 65.35, indicates that in 1990 about 65.35% of U.S. households with TV had basic cable service.

LESSON 12.1 • Right Triangle Trigonometry

1. a. \( \sin A = \frac{3}{5} \), \( \cos A = \frac{4}{5} \), \( \tan A = \frac{3}{4} \), \( \sin B = \frac{4}{5} \), \( \cos B = \frac{3}{5} \), \( \tan B = \frac{4}{3} \)
b. \( \sin A = \frac{12}{13} \), \( \cos A = \frac{5}{13} \), \( \tan A = \frac{12}{5} \), \( \sin B = \frac{5}{13} \), \( \cos B = \frac{12}{13} \), \( \tan B = \frac{5}{12} \)

2. a. \( r = 7.6 \)    b. \( S \approx 42.0^\circ \)

c. \( z \approx 7.8 \)

3. a. \( \sin 29^\circ = \frac{b}{35} \), \( b \approx 17 \)    b. \( \tan 43^\circ = \frac{a}{6.9} \), \( a \approx 6.4 \)

c. \( \cos B = \frac{5.5}{8.8} \), \( B \approx 51^\circ \)

LESSON 12.2 • The Law of Sines

1. a. \( \frac{9 \sin 20^\circ}{\sin 75^\circ} = 3.2 \)    b. \( \frac{6.2 \sin 95^\circ}{\sin 45^\circ} \approx 8.7 \)

c. \( \frac{12 \sin 120.5^\circ}{\sin 32.4^\circ} \approx 19.3 \)

2. a. \( A = 72.7^\circ \), \( b = 7.2 \) cm, \( a = 7.6 \) cm

b. \( B = 25^\circ \), \( a = 23.8 \) mm, \( c = 37.1 \) mm

3. a. 2

b. 0

4. a. \( \frac{9 \sin 45^\circ}{\sin 142^\circ} \approx 1.4 \)

b. 18.2 km    c. 24.0 km

LESSON 12.3 • The Law of Cosines

1. a. \( b = 38.5 \)    b. \( C \approx 47.5^\circ \)

2. a. \( a^2 = 9^2 + 12^2 - 2(9)(12) \cos 110^\circ \); \( a = \sqrt{225 - 216 \cos 110^\circ} \approx 17.3 \)

b. \( b^2 = 3^2 + 4^2 - 2(3)(4) \cos A \); \( A = 90^\circ \)

3. a. Law of Sines    b. Law of Cosines

4. a. \( c = 17.4 \) cm, \( B = 9.3^\circ \), \( A = 22.7^\circ \)

b. \( A = 30.2^\circ \), \( B = 83.6^\circ \), \( C = 66.2^\circ \). (The sum of these angle measures is 179.9°, rather than 180°, due to rounding.)

5. 88.4°

LESSON 12.4 • Extending Trigonometry

1. a.

b.
c. \[200° ; 20°\]

2. a. \(-\frac{\sqrt{2}}{2}\)  
   b. \(-2\)

3. a. negative  
   b. negative  
   c. positive  
   d. negative  
   e. positive  
   f. negative

LESSON 12.5 • Introduction to Vectors

1. a. \((-1, 1)\)  
   b. \((3, 3)\)  
   c. \(\sqrt{26}\)  
   d. \((0, 2)\)

2. a. \((\sqrt{2}, \sqrt{2})\)  
   b. \((0, 2)\)  
   c. \((0, -5)\)

3. a. \((2 \angle 180°)\)  
   b. \((5 \angle 53°)\)  
   c. \((\sqrt{5} \angle 207°)\)

4. a. Dock 2  
   b. 74.4 miles; 6.2°

LESSON 12.6 • Parametric Equations

1. a.  
<table>
<thead>
<tr>
<th>(t)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>-1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

   b.  
<table>
<thead>
<tr>
<th>(t)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>7</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>7</td>
</tr>
</tbody>
</table>

2. a. \(y = 3x^2 + 2x - 1\)  
   b. \(y = \pm 3\sqrt{x} - 2\)  
   c. \(y = \pm 3\sqrt{2x} - 4 - 5\)

3.

LESSON 12.7 • Radian Measure and Arc Length

1. a. 225°  
   b. \(\frac{\pi}{12}\)  
   c. \(\frac{11\pi}{6}\)
   d. \(-120°\)  
   e. \(-\frac{7\pi}{9}\)  
   f. \(-\frac{13\pi}{3}\)
   g. \(-330°\)  
   h. 204°

2. a. 10π  
   b. 13.5  
   c. \(\frac{\pi}{8}\)

3. a. \(\theta = 120°\)  
   b. \(\theta = 270°\)  
   c. \(\theta = \frac{5\pi}{3}\)  
   d. \(\theta = \frac{\pi}{6}\)

4. a. 427.3 mm  
   b. 8.9 mm/min  
   c. 0.105 radians/min

4. a. \(x = 6.5t \cos 0°, y = -16t^2 + 6.5t \sin 0° + 75,\)  
   or \(x = 6.5t, y = -16t^2 + 75\)  
   b. \(-16t^2 + 75 = 0\)  
   c. 2.17 s; 14.11 ft

LESSON 13.1 • Defining the Circular Functions

1. a. \(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\)  
   b. \(-\frac{1}{2}\)  
   c. \(-\frac{1}{2}\)  
   d. 0

2. a. 0.6018; 37°  
   b. -0.4226; 65°

3.

LESSON 13.2 • Radian Measure and Arc Length

1. a. 225°  
   b. \(\frac{\pi}{12}\)  
   c. \(\frac{11\pi}{6}\)
   d. \(-120°\)  
   e. \(-\frac{7\pi}{9}\)  
   f. \(-\frac{13\pi}{3}\)
   g. \(-330°\)  
   h. 204°

2. a. 10π  
   b. 13.5  
   c. \(\frac{\pi}{8}\)

3. a. \(\theta = 120°\)  
   b. \(\theta = 270°\)  
   c. \(\theta = \frac{5\pi}{3}\)  
   d. \(\theta = \frac{\pi}{6}\)

4. a. 427.3 mm  
   b. 8.9 mm/min  
   c. 0.105 radians/min
LESSON 13.3 • Graphing Trigonometric Functions

1. Possible answers:
   a. \( y = -1 + 3 \cos(2x) \); amplitude: 3; period: \( \pi \);
      phase shift: none; vertical shift: 1 unit down
   b. \( y = -\sin x + 3 \); amplitude: 1; period: \( 2\pi \);
      phase shift: none; vertical shift: 3 units up

2. a. 
   ![Graph of trig function]

   b. 
   ![Graph of trig function]

   c. 
   ![Graph of trig function]

3. a. \( y = 2.5 \cos\left(x - \frac{\pi}{4}\right) \)
   
   b. \( y = 5 + 3 \sin \frac{4}{3}x \), or \( y = 3 \sin \frac{4}{3}x + 5 \)

LESSON 13.4 • Inverses of Trigonometric Functions

1. a. \( 36.7^\circ; 0.64 \)
   c. \( 29.9^\circ; 0.52 \)
   d. \(-42.8^\circ; -0.75 \)

2. a. \( \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{-5\pi}{3} \)
   
   b. \( \frac{5\pi}{12}, \frac{19\pi}{12}, \frac{-5\pi}{12}, \frac{-19\pi}{12} \)
   
   c. \( 1.25, 1.89, -4.39, -5.03 \)
   d. \( 0.73, 5.55, -0.73, -5.55 \)
   e. \( \frac{3\pi}{5}, \frac{7\pi}{5}, \frac{-3\pi}{5}, \frac{-7\pi}{5} \)
   
   f. \( -\frac{5\pi}{6}, -\frac{\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \)

3. a. \( x = 1.385 \) and \( x = 1.757 \)
   
   b. \( x = 0.766 \) and \( x = 5.517 \)

LESSON 13.5 • Modeling with Trigonometric Equations

1. a. \( x = \left\{ \frac{\pi}{2} \right\} \)
   
   b. \( x = \left\{ \frac{3\pi}{2} \right\} \)
   
   c. \( x = \left\{ \frac{5\pi}{9}, \frac{11\pi}{9}, \frac{13\pi}{9}, \frac{17\pi}{9} \right\} \)
   
   d. \( x = \left\{ \frac{5\pi}{3} \right\} \)
   
   e. \( x = \left\{ \frac{3\pi}{2} \right\} \)
   
   f. \( x = \left\{ \frac{5\pi}{12}, \frac{11\pi}{12} \right\} \)

2. a. \( x = \{0.12, 5.36\} \)
   
   b. \( x = \{1.89, 2.82, 5.03, 5.96\} \)

3. a. 8.5  b. 8.5  c. 5  d. 3.5
   
   e. 13.5  f. 5  g. \( \frac{7\pi}{2\pi} \)  h. 7
   
   i. 4  j. 4

4. a. About 14.4 h (or 14 h 24 min)
   
   b. Days 105 and 238

LESSON 13.6 • Fundamental Trigonometric Identities

1. a. \( \sqrt{3} \)
   
   b. \( -\sqrt{3} \)
   
   c. \( \sqrt{2} \)
   
   d. \( \frac{-2}{\sqrt{3}}, \text{ or } -\frac{2\sqrt{3}}{3} \)
   
   e. Undefined
   
   f. \(-2 \)

2. Possible answers:
   
   a. \( y = \tan x \)
   
   b. \( y = \sin x \)
   
   c. \( y = -\csc x \)

3. a. \( \frac{\sin \theta + 1}{\cos \theta} \)
   
   b. \( \cos^2 \theta, \text{ or } 1 - \sin^2 \theta \)
   
   c. 0
   
   d. 1

4. a. Not an identity
   
   b. Identity
   
   c. Identity
   
   d. Identity

LESSON 13.7 • Combining Trigonometric Functions

1. a. Identity
   
   b. Not an identity
   
   c. Identity
   
   d. Not an Identity

2. \( \sin 3A = \sin(2A + A) = \sin 2A \cos A + \cos 2A \sin A \)
   
   \( = (2\sin A \cos A)\cos A + (\cos^2 A - \sin^2 A)\sin A \)
   
   \( = 2\sin A \cos^2 A + \cos^2 A \sin A - \sin^3 A \)

3. a. \( \sin 0.7 \)
   
   b. \( \sin 9.6 \)
   
   c. \( \cos 1.6 \)
   
   d. \( \cos(-1.5), \text{ or } \cos 1.5 \)

4. a. \( \sqrt{2} - \sqrt{6} \)
   
   b. \( \sqrt{2} + \sqrt{6} \)
   
   c. \( \sqrt{6} - \sqrt{2} \)

5. a. \( \sin 2x = \frac{24}{25} \)
   
   b. \( \cos 2x = \frac{7}{25} \)
   
   c. \( \tan 2x = \frac{24}{7} \)

b. \( \sin 2x =-\frac{120}{169} \)

ANSWERS 103
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