PLTW Engineering Formula Sheet 2017 (v17.0)

1.0 Statistics

Mean

$$\mu = \frac{\sum x_i}{N} \quad (1.1a)$$

$$\bar{x} = \frac{\sum x_i}{n} \quad (1.1b)$$

μ = population mean
$\bar{x}$ = sample mean
$\sum x_i$ = sum of all data values (x_1, x_2, x_3, …)
N = size of population
n = size of sample

Median

Place data in ascending order.
If N is odd, median = central value
If N is even, median = mean of two central values (1.2)

N = size of population

Range (1.5)

Range = x_{max} - x_{min} (1.3)

x_{max} = maximum data value
x_{min} = minimum data value

Mode

Place data in ascending order.
Mode = most frequently occurring value (1.4)

If two values occur with maximum frequency the data set is *bimodal.*
If three or more values occur with maximum frequency the data set is *multi-modal.*

Standard Deviation

$$\sigma = \sqrt{\frac{\sum (x_i - \mu)^2}{N}} \quad \text{(Population)} \quad (1.5a)$$

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} \quad \text{(Sample)} \quad (1.5b)$$

σ = population standard deviation
s = sample standard deviation
x_i = individual data value (x_1, x_2, x_3, …)
μ = population mean
$\bar{x}$ = sample mean
N = size of population
n = size of sample

2.0 Probability

Frequency

$$f_x = \frac{n_x}{n} \quad (2.1)$$

f_x = relative frequency of outcome x
n_x = number of events with outcome x
n = total number of events

Binomial Probability (order doesn't matter)

$$P_k = \frac{n!(p^k)(q^{n-k})}{k!(n-k)!} \quad (2.2)$$

P_k = binomial probability of k successes in n trials
p = probability of a success
q = 1 – p = probability of failure
k = number of successes
n = number of trials

Independent Events

P (A and B and C) = $P_A P_B P_C$ (2.3)

P (A and B and C) = probability of independent events A and B and C occurring in sequence
P_A = probability of event A

Mutually Exclusive Events

P (A or B) = P_A + P_B (2.4)

P (A or B) = probability of either mutually exclusive event A or B occurring in a trial
P_A = probability of event A

Conditional Probability

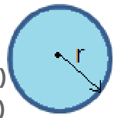
$$P(A|D) = \frac{P(A)\cdot P(D|A)}{P(A)\cdot P(D|A) + P(\sim A)\cdot P(D|\sim A)} \quad (2.5)$$

P (A|D) = probability of event A given event D
P(A) = probability of event A occurring
P(~A) = probability of event A not occurring
P(D|~ A) = probability of event D given event A did not occur

3.0 Plane Geometry

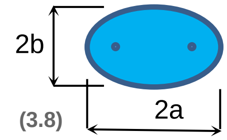
Circle

Circumference $= 2 \pi r$ (3.1)
Area $= \pi r^2$ (3.2)



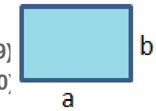
Ellipse

Area $= \pi a b$ (3.8)



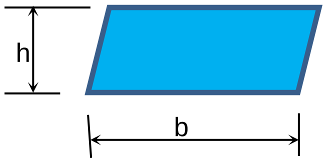
Rectangle

Perimeter $= 2a + 2b$ (3.9)
Area $= ab$ (3.10)



Parallelogram

Area $= bh$ (3.3)



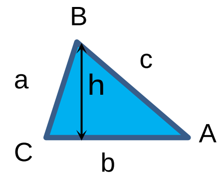
Triangle (3.6)

Area $= \frac{1}{2} bh$ (3.11)

$a^2 = b^2 + c^2 - 2bc \cdot \cos\angle A$ (3.12)

$b^2 = a^2 + c^2 - 2ac \cdot \cos\angle B$ (3.13)

$c^2 = a^2 + b^2 - 2ab \cdot \cos\angle C$ (3.14)



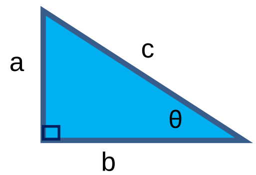
Right Triangle

$c^2 = a^2 + b^2$ (3.4)

$\sin\theta = \frac{a}{c}$ (3.5)

$\cos\theta = \frac{b}{c}$ (3.6)

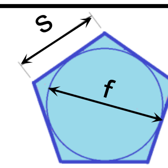
$\tan\theta = \frac{a}{b}$ (3.7)



Regular Polygons

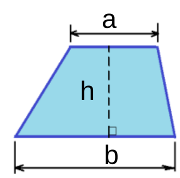
$$\text{Area} = n\frac{s(\frac{1}{2}f)}{2} = \frac{ns^2}{4\tan\left(\frac{180}{n}\right)} \quad (3.15)$$

n = number of sides



Trapezoid

Area $= \frac{1}{2}(a + b)h$ (3.16)

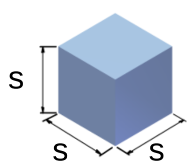


4.0 Solid Geometry

Cube

Volume $= s^3$ (4.1)

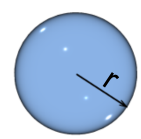
Surface Area $= 6s^2$ (4.2)



Sphere

Volume $= \frac{4}{3} \pi r^3$ (4.8)

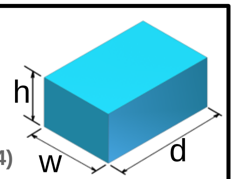
Surface Area $= 4 \pi r^2$ (4.9)



Rectangular Prism

Volume = wdh (4.3)

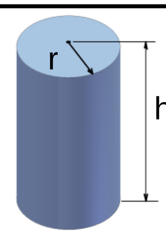
Surface Area = 2(wd + wh + dh) (4.4)



Cylinder

Volume $= \pi r^2 h$ (4.10)

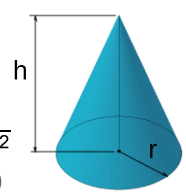
Surface Area $= 2 \pi r h + 2 \pi r^2$ (4.11)



Right Circular Cone

Volume $= \frac{\pi r^2 h}{3}$ (4.5)

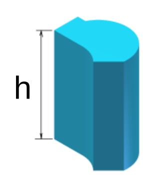
Total Surface Area $= \pi r^2 + \pi r \sqrt{r^2 + h^2}$ (4.6)



Irregular Prism

Volume = Ah (4.12)

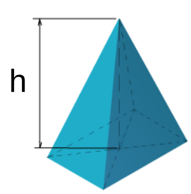
A = area of base



Pyramid

Volume $= \frac{Ah}{3}$ (4.7)

A = area of base



5.0 Constants

$g = 9.8\ m/s^2 = 32.17\ ft/s^2$

$G = 6.67 \times 10^{-11}\ m^3/kg \cdot s^2$

$\pi = 3.14159$

6.0 Conversions

Mass/Weight (6.1)

1 kg = 2.205 lb_m
1 slug = 32.2 lb_m
1 ton = 2000 lb
1 lb = 16 oz

Length (6.2)

1 m = 3.28 ft
1 km = 0.621 mi
1 in. = 2.54 cm
1 mi = 5280 ft
1 yd = 3 ft

Time (6.3)

1 d = 24 h
1 h = 60 min
1 min = 60 s
1 yr = 365 d

Area (6.4)

1 acre = 4047 m^2
= 43,560 ft^2
= 0.00156 mi^2

Volume (6.5)

1L = 0.264 gal
= 0.0353 ft^3
= 33.8 fl oz
1mL = 1 cm^3 = 1 cc

Temperature Unit Equivalents (6.6)
*Use equation in section 9.0 to convert

Δ1 K = Δ 1 ºC
= Δ 1.8 ºF
= Δ 1.8 ºR

Force (6.7)

1 N = 0.225 lb
1 kip = 1,000 lb

Pressure (6.8)

1 atm = 1.01325 bar
= 33.9 ft H_2O
= 29.92 in. Hg
= 760 mm Hg
= 101,325 Pa
= 14.7 psi
1psi = 2.31 ft of H_2O

Power (6.9)

1 W = 3.412 Btu/h
= 0.00134 hp
= 14.34 cal/min
= 0.7376 ft·lb_f/s
1 hp = 550 ft·lb/sec

Energy (6.10)

1 J = 0.239 cal
= 9.48 x 10^{-4} Btu
= 0.7376 ft·lb_f
1kW h = 3,600,000 J

Rotational Speed (6.11)

1 Hz = 2π rad/sec
= 60 rpm

7.0 Defined Units

1 J = 1 N·m
1 N = 1 kg·m / s^2
1 Pa = 1 N / m^2
1 V = 1 W / A
1 W = 1 J / s
1 Ω = 1 V / A
1 Hz = 1 s^{-1}
1 F = 1 A·s / V
1 H = 1 V·s / V

8.0 SI Prefixes

Numbers Less Than One

Power of 10	Decimal Equivalent	Prefix	Abbreviation
10^{-1}	0.1	deci-	d
10^{-2}	0.01	centi-	c
10^{-3}	0.001	milli-	m
10^{-6}	0.000001	micro-	µ
10^{-9}	0.000000001	nano-	n
10^{-12}		pico-	p
10^{-15}		femto-	f
10^{-18}		atto-	a
10^{-21}		zepto-	z
10^{-24}		yocto-	y

Numbers Greater Than One

Power of 10	Whole Number Equivalent	Prefix	Abbreviation
10^{1}	10	deca-	da
10^{2}	100	hecto-	h
10^{3}	1000	kilo-	k
10^{6}	1,000,000	Mega-	M
10^{9}	1,000,000,000	Giga-	G
10^{12}		Tera-	T
10^{15}		Peta-	P
10^{18}		Exa-	E
10^{21}		Zetta-	Z
10^{24}		Yotta-	Y

9.0 Equations

Mass and Weight

$m = VD_m$ (9.1)

$W = mg$ (9.2)

$W = VD_w$ (9.3)

V = volume
D_m = mass density
m = mass
D_w = weight density
W = weight
g = acceleration due to gravity

Temperature

$T_K = T_C + 273$ (9.4)

$T_R = T_F + 460$ (9.5)

$T_F = \frac{9}{5} T_C + 32$ (9.6a)

$T_C = \frac{T_F - 32}{1.8}$ (9.6b)

T_K = temperature in Kelvin
T_C = temperature in Celsius
T_R = temperature in Rankin
T_F = temperature in Fahrenheit

Force and Moment

$F = ma$ (9.7a) $M = Fd_\perp$ (9.7b)

F = force
m = mass
a = acceleration
M = moment
$d_\perp$ = perpendicular distance

Equations of Static Equilibrium

$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma M_P = 0$ (9.8)

F_x = force in the x-direction
F_y = force in the y-direction
M_P = moment about point P

9.0 Equations (Continued)

Energy: Work

$W = F_{\parallel} \cdot \mathbf{d}$ (9.9)

W = work
$F_{\parallel}$ = force parallel to direction of displacement
d = displacement

Power

$P = \frac{E}{t} = \frac{W}{t}$ (9.10)

$P = \tau\,\omega$ (9.11)

P = power
E = energy
W = work
t = time
τ = torque
ω = angular velocity

Efficiency

$\text{Efficiency (\%)} = \frac{P_{out}}{P_{in}} \cdot 100\%$ (9.12)

P_{out} = useful power output
P_{in} = total power input

Energy: Potential

$U = mgh$ (9.13)

U = potential energy
m =mass
g = acceleration due to gravity
h = height

Energy: Kinetic

$K = \frac{1}{2} mv^2$ (9.14)

K = kinetic energy
m = mass
v = velocity

Energy: Thermal

$\Delta Q = mc\Delta T$ (9.15)

ΔQ = change in thermal energy
m = mass
c = specific heat
ΔT = change in temperature

Fluid Mechanics

$p = \frac{F}{A}$ (9.16)

$\frac{V_1}{T_1} = \frac{V_2}{T_2}$ (Charles' Law) (9.17)

$\frac{p_1}{T_1} = \frac{p_2}{T_2}$ (Gay-Lussanc's Law) (9.18)

$p_1V_1 = p_2V_2$ (Boyle's Law) (9.19)

$Q = Av$ (9.20)

$A_1v_1 = A_2v_2$ (9.21)

$P = Qp$ (9.22)

absolute pressure = gauge pressure + atmospheric pressure (9.23)

p = absolute pressure
F = force
A = area
V = volume
T = absolute temperature
Q = flow rate
v = flow velocity
P = power

Mechanics

$\bar{s} = \frac{d}{t}$ (9.24)

$\bar{\mathbf{v}} = \frac{\Delta \mathbf{d}}{\Delta t}$ (9.25)

$a = \frac{v_f - v_i}{t}$ (9.26)

$X = \frac{v_i^2 \sin(2\theta)}{-g}$ (9.27)

$v = v_i + at$ (9.28)

$d = d_i + v_it + \frac{1}{2}at^2$ (9.29)

$v^2 = v_i^2 + 2a(d - d_i)$ (9.30)

$\tau = dF\sin\theta$ (9.31)

$\bar{s}$ = average speed
$\bar{\mathbf{v}}$ = average velocity
v = velocity
v_i = initial velocity (t =0)
a = acceleration
X = range
t = time
Δd = change in displacement
d = distance
d_i = initial distance (t=0)
g = acceleration due to gravity
θ = angle
τ = torque
F = force

Electricity

Ohm's Law

$V = IR$ (9.32)

$P = IV$ (9.33)

$R_T \text{ (series)} = R_1 + R_2 + \cdots + R_n$ (9.34)

$R_T \text{ (parallel)} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$ (9.35)

Kirchhoff's Current Law

$I_T = I_1 + I_2 + \cdots + I_n$
or $I_T = \sum_{k=1}^{n} I_k$ (9.36)

Kirchhoff's Voltage Law

$V_T = V_1 + V_2 + \cdots + V_n$
or $V_T = \sum_{k=1}^{n} V_k$ (9.37)

V = voltage
V_T = total voltage
I = current
I_T = total current
R = resistance
R_T = total resistance
P = power

Thermodynamics

$P = Q' = AU\Delta T$ (9.38)

$P = Q' = \frac{\Delta Q}{\Delta t}$ (9.39)

$U = \frac{1}{R} = \frac{k}{L}$ (9.40)

$P = \frac{kA\Delta T}{L}$ (9.41)

$A_1v_1 = A_2v_2$ (9.42)

$P_{net} = \sigma Ae(T_2^4 - T_1^4)$ (9.43)

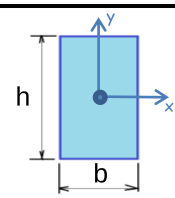
$k = \frac{PL}{A\Delta T}$ (9.44)

P = rate of heat transfer
Q = thermal energy
A = area of thermal conductivity
U = coefficient of heat conductivity (U-factor)
ΔT = change in temperature
Δt = change in time
R = resistance to heat flow (R-value)
k = thermal conductivity
v = velocity
P_{net} = net power radiated
$\sigma = 5.6696 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$
e = emissivity constant
L = thickness
T_1, T_2 = temperature at time 1, time 2

10.0 Section Properties

Moment of Inertia

$$I_{xx} = \frac{bh^3}{12} \qquad (10.1)$$



I_{xx} = moment of inertia of a rectangular section about x axis

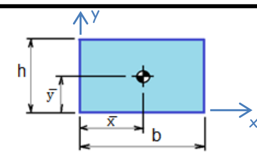
Complex Shapes Centroid

$$\bar{x} = \frac{\Sigma x_i A_i}{\Sigma A_i} \text{ and } \bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i} \qquad (10.2)$$

$\bar{x}$ = x-distance to the centroid
$\bar{y}$ = y-distance to the centroid
x_i = x distance to centroid of shape i
y_i = y distance to centroid of shape i
A_i = Area of shape i

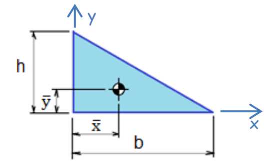
Rectangle Centroid

$$\bar{x} = \frac{b}{2} \text{ and } \bar{y} = \frac{h}{2} \qquad (10.3)$$



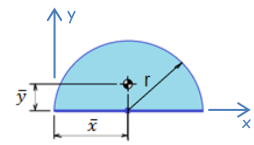
Right Triangle Centroid

$$\bar{x} = \frac{b}{3} \text{ and } \bar{y} = \frac{h}{3} \qquad (10.4)$$



Semi-circle Centroid

$$\bar{x} = r \text{ and } \bar{y} = \frac{4r}{3\pi} \qquad (10.5)$$



$\bar{x}$ = x-distance to the centroid
$\bar{y}$ = y-distance to the centroid

11.0 Material

Stress (axial)

$$\sigma = \frac{F}{A} \qquad (11.1)$$

σ = stress
F = axial force
A = cross-sectional area

Strain (axial)

$$\varepsilon = \frac{\delta}{L_0} \qquad (11.2)$$

ε = strain
L_0 = original length
δ = change in length

Modulus of Elasticity

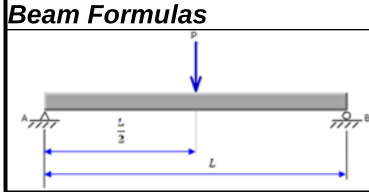
$$E = \frac{\sigma}{\varepsilon} \qquad (11.3)$$

$$E = \frac{(F_2-F_1)L_0}{(\delta_2-\delta_1)A} \qquad (11.4)$$

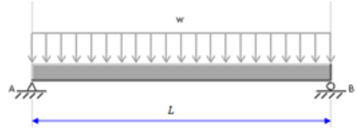
E = modulus of elasticity
σ = stress
ε = strain
A = cross-sectional area
F = axial force
δ = deformation

12.0 Structural Analysis

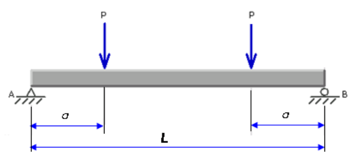
Beam Formulas



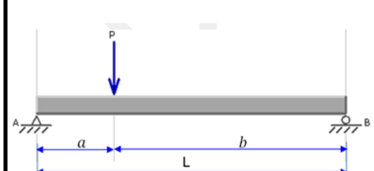
Reaction $R_A = R_B = \frac{P}{2}$ (12.1)
Moment $M_{max} = \frac{PL}{4}$ (at point of load) (12.2)
Deflection $\Delta_{max} = \frac{PL^3}{48EI}$ (at point of load) (12.3)



Reaction $R_A = R_B = \frac{\omega L}{2}$ (12.4)
Moment $M_{max} = \frac{\omega L^2}{8}$ (at center) (12.5)
Deflection $\Delta_{max} = \frac{5\omega L^4}{384EI}$ (at center) (12.6)



Reaction $R_A = R_B = P$ (12.7)
Moment $M_{max} = Pa$ (12.8)
Deflection $\Delta_{max} = \frac{Pa}{24EI}(3L^2-4a^2)$ (at center) (12.9)



Reaction $R_A = \frac{Pb}{L}$ and $R_B = \frac{Pa}{L}$ (12.10)
Moment $M_{max} = \frac{Pab}{L}$ (at Point of Load) (12.11)
Deflection $\Delta_{max} = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EIL}$ (12.12)
(at $x = \sqrt{\frac{a(a+2b)}{3}}$ when $a>b$)

E = modulus of elasticity
I = moment of inertia

Deformation: Axial

$$\delta = \frac{FL_0}{AE} \qquad (12.13)$$

δ = deformation
F = axial force
L_0 = original length
A = cross-sectional area
E = modulus of elasticity

Truss Analysis

2J = M + R (12.14)

J = number of joints
M =number of members
R = number of reaction forces

13.0 Simple Machines

Mechanical Advantage (MA)

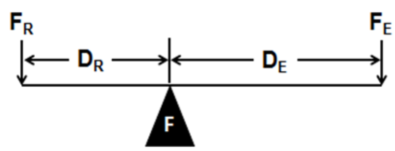
$$IMA = \frac{D_E}{D_R} \quad (13.1) \qquad AMA = \frac{F_R}{F_E} \quad (13.2)$$

$$\%\ Efficiency = \left(\frac{AMA}{IMA}\right) 100 \quad (13.3)$$

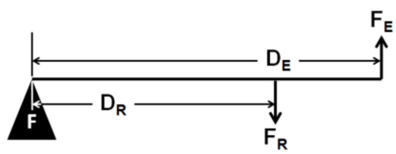
IMA = ideal mechanical advantage
AMA = actual mechanical advantage
D_E = effort distance
D_R = resistance distance
F_E = effort force
F_R = resistance force

Lever

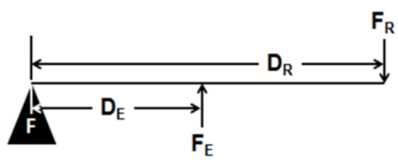
1st Class



2nd Class

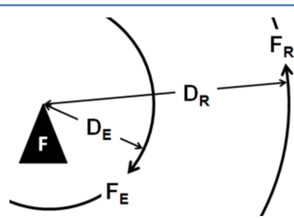


3rd Class

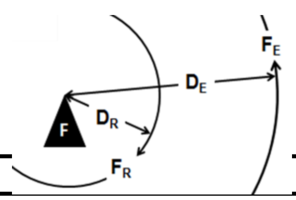


Wheel and Axle

Effort at Axle



Effort at Wheel



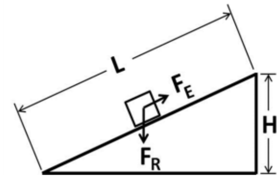
Pulley Systems

IMA = total number of strands of a single string supporting the resistance (13.4)

$$IMA = \frac{D_E \text{ (string pulled)}}{D_R \text{ (resistance lifted)}} \quad (13.5)$$

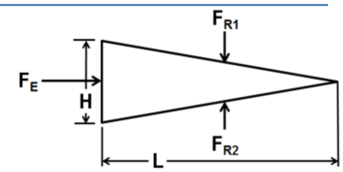
Inclined Plane

$$IMA = \frac{L}{H} \quad (13.6)$$



Wedge

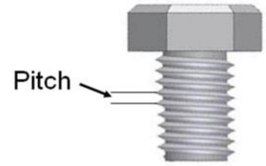
$$IMA = \frac{L}{H} \quad (13.7)$$

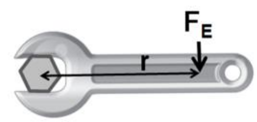


Screw

$$IMA = \frac{C}{Pitch} \quad (13.8)$$

$$Pitch = \frac{1}{TPI} \quad (13.9)$$





C = circumference
r = radius
Pitch = distance between threads
TPI = threads per inch

Compound Machines

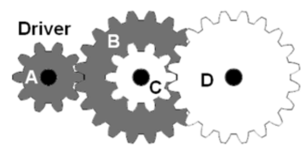
$$MA_{TOTAL} = (MA_1)(MA_2)(MA_3)\ldots \quad (13.10)$$

Gears; Sprockets with Chains; and Pulleys with Belts Ratios

$$GR = \frac{N_{out}}{N_{in}} = \frac{d_{out}}{d_{in}} = \frac{\omega_{in}}{\omega_{out}} = \frac{\tau_{out}}{\tau_{in}} \quad (13.11)$$

$$\frac{d_{out}}{d_{in}} = \frac{\omega_{in}}{\omega_{out}} = \frac{\tau_{out}}{\tau_{in}} \text{ (pulleys)} \quad (13.12)$$

Compound Gears



$$GR_{TOTAL} = \left(\frac{B}{A}\right)\left(\frac{D}{C}\right) \quad (13.13)$$

GR = gear ratio
ω_{in} = angular velocity - driver
ω_{out} = angular velocity - driven
N_{in} = number of teeth - driver
N_{out} = number of teeth - driven
d_{in} = diameter - driver
d_{out} = diameter - driven
τ_{in} = torque - driver
τ_{out} = torque - driven

14.0 Structural Design

Steel Beam Design: Shear

$V_a \leq \frac{V_n}{\Omega_v}$ (14.1)

$V_n = 0.6 F_y A_w$ (14.2)

V_a = internal shear force
V_n = nominal shear strength
Ω_v = 1.5 = factor of safety for shear
F_y = yield stress
A_w = area of web
$\frac{V_n}{\Omega_v}$ = allowable shear strength

Steel Beam Design: Moment

$M_a \leq \frac{M_n}{\Omega_b}$ (14.3)

$M_n = F_y Z_x$ (14.4)

M_a = internal bending moment
M_n = nominal moment strength
Ω_b = 1.67 = factor of safety for bending moment
F_y = yield stress
Z_x = plastic section modulus about neutral axis
$\frac{M_n}{\Omega_b}$ = allowable bending strength

Spread Footing Design

$q_{net} = q_{allowable} - p_{footing}$ (14.5)

$p_{footing} = t_{footing} \cdot 150 \frac{lb}{ft^3}$ (14.6)

$q = \frac{P}{A}$ (14.7)

q_{net} = net allowable soil bearing pressure
$q_{allowable}$ = total allowable soil bearing pressure
$p_{footing}$ = soil bearing pressure due to footing weight
$t_{footing}$ = thickness of footing
q = soil bearing pressure
P = column load applied
A = area of footing

15.0 Storm Water Runoff

Storm Water Drainage

$Q = C_f C i A$ (15.1)

$C_c = \frac{C_1 A_1 + C_2 A_2 + \cdots}{A_1 + A_2 + \cdots}$ (15.2)

Q = peak storm water runoff rate (ft^3/s)
C_f = runoff coefficient adjustment factor
C = runoff coefficient
i = rainfall intensity (in./h)
A = drainage area (acres)

Runoff Coefficient Adjustment Factor	
Return Period	***Cf***
1, 2, 5, 10	***1.0***
25	***1.1***
50	***1.2***
100	***1.25***

Rational Method Runoff Coefficients	
Categorized by Surface	
Forested	0.059—0.2
Asphalt	0.7—0.95
Brick	0.7—0.85
Concrete	0.8—0.95
Shingle roof	0.75—0.95
Lawns, well drained (sandy soil)	
Up to 2% slope	0.05—0.1
2% to 7% slope	0.10—0.15
Over 7% slope	0.15—0.2
Lawns, poor drainage (clay soil)	
Up to 2% slope	0.13—0.17
2% to 7% slope	0.18—0.22
Over 7% slope	0.25—0.35
Driveways,	0.75—0.85
Categorized by Use	
Farmland	0.05—0.3
Pasture	0.05—0.3
Unimproved	0.1—0.3
Parks	0.1—0.25
Cemeteries	0.1—0.25
Railroad yard	0.2—0.40
Playgrounds	0.2—0.35
Business Districts	
Neighborhood	0.5—0.7
City (downtown)	0.7—0.95
Residential	
Single-family	0.3—0.5
Multi-plexes,	0.4—0.6
Multi-plexes,	0.6—0.75
Suburban	0.25—0.4
Apartments,	0.5—0.7
Industrial	
Light	0.5—0.8
Heavy	0.6—0.9

16.0 Water Supply

Hazen-Williams Formula

$h_f = \frac{10.44 L Q^{1.85}}{C^{1.85} d^{4.8655}}$ (16.1)

h_f = head loss due to friction (ft of H_2O)
L = length of pipe (ft)
Q = water flow rate (gpm)
C = Hazen-Williams constant
d = diameter of pipe (in.)

Dynamic Head

dynamic head = static head – head loss (16.2)
static head = change in elevation between source and discharge

17.0 Heat Loss/Gain

Heat Loss/Gain

$Q' = A U \Delta T$ (17.1)

$U = \frac{1}{R}$ (17.2)

Q = thermal energy
A = area of thermal conductivity
U = coefficient of heat conductivity (U-factor)
ΔT = change in temperature
R = resistance to heat flow (R-value)

18.0 Hazen-Williams Constants

Pipe Material	Typical Range	Clean, New Pipe	Typical Design Value
Cast Iron and Wrought Iron	80 - 150	130	100
Copper, Glass or Brass	120 - 150	140	130
Cement lined Steel or Iron		150	140
Plastic PVC or ABS	120 - 150	140	130
Steel, welded and seamless or interior riveted	80-150	140	100

19.0 Equivalent Length of (Generic) Fittings

Screwed Fittings		Pipe Size										
		1/4	3/8	1/2	3/4	1	1 ¼	1 ½	2	2 ½	3	4
Elbows	Regular 90 degree	2.3	3.1	3.6	4.4	5.2	6.6	7.4	8.5	9.3	11.0	13.0
	Long radius 90 degree	1.5	2.0	2.2	2.3	2.7	3.2	3.4	3.6	3.6	4.0	4.6
	Regular 45 degree	0.3	0.5	0.7	0.9	1.3	1.7	2.1	2.7	3.2	4.0	5.5
Tees	Line Flow	0.8	1.2	1.7	2.4	3.2	4.6	5.6	7.7	9.3	12.0	17.0
	Branch Flow	2.4	3.5	4.2	5.3	6.6	8.7	9.9	12.0	13.0	17.0	21.0
Return	Regular 180 degree	2.3	3.1	3.6	4.4	5.2	6.6	7.4	8.5	9.3	11.0	13.0
Valves	Globe	21.0	22.0	22.0	24.0	29.0	37.0	42.0	54.0	62.0	79.0	110.0
	Gate	0.3	0.5	0.6	0.7	0.8	1.1	1.2	1.5	1.7	1.9	2.5
	Angle	12.8	15.0	15.0	15.0	17.0	18.0	18.0	18.0	18.0	18.0	18.0
	Swing Check	7.2	7.3	8.0	8.8	11.0	13.0	15.0	19.0	22.0	27.0	38.0
Strainer			4.6	5.0	6.6	7.7	18.0	20.0	27.0	29.0	34.0	42.0

Flanged Fittings		Pipe Size																
		1/2	3/4	1	1 ¼	1 ½	2	2 ½	3	4	5	6	8	10	12	14	16	18
Elbows	Regular 90 degree	0.9	1.2	1.6	2.1	2.4	3.1	3.6	4.4	5.9	7.3	8.9	12.0	14.0	17.0	18.0	21.0	23.0
	Long radius 90	1.1	1.3	1.6	2.0	2.3	2.7	2.7	3.4	4.2	5.0	5.7	7.0	8.0	9.0	9.4	10.0	11.0
	Regular 45 degree	0.5	0.6	0.8	1.1	1.3	1.7	2.0	2.5	3.5	4.5	5.6	7.7	9.0	11.0	13.0	15.0	16.0
Tees	Line Flow	0.7	0.8	1.0	1.3	1.5	1.8	1.9	2.2	2.8	3.3	3.8	4.7	5.2	6.0	6.4	7.2	7.6
	Branch Flow	2.0	2.6	3.3	4.4	5.2	6.6	7.5	9.4	12.0	15.0	18.0	24.0	30.0	34.0	37.0	43.0	47.0
Return Bends	Regular 180 degree	0.9	1.2	1.6	2.1	2.4	3.1	3.6	4.4	5.9	7.3	8.9	12.0	14.0	17.0	18.0	21.0	23.0
	Long radius 180	1.1	1.3	1.6	2.0	2.3	2.7	2.9	3.4	4.2	5.0	5.7	7.0	8.0	9.0	9.4	10.0	11.0
Valves	Globe	38.0	40.0	45.0	54.0	59.0	70.0	77.0	94.0	120.0	150.0	190.0.	260.0	310.0	390.0			
	Gate						2.6	2.7	2.8	2.9	3.1	3.2	3.2	3.2	3.2	3.2	3.2	3.2
	Angle	15.0	15.0	17.0	18.0	18.0	21.0	22.0	285.0	38.0	50.0	63.0	90.0	120.0	140.0	160.0	190.0	210.0
	Swing Check	3.8	5.3	7.2	10.0	12.0	17.0	21.0	27.0	38.0	50.0	63.0	90.0	120.0	140.0			

20.0 555 Timer Design

$T = 0.693\,(R_A + 2R_B)C$ (20.1)

$f = \frac{1}{T}$ (20.2)

$\text{duty-cycle} = \frac{(R_A + R_B)}{(R_A + 2R_B)} \cdot 100\%$ (20.3)

T = period
f = frequency
R_A = resistance A
R_B = resistance B
C = capacitance

21.B Resistor Color Code

	1st Band	2nd Band	Multiplier	Tolerance
NONE				20%
Silver			0.01	10%
Gold			0.1	5%
Black	0	0	1	
Brown	1	1	10	
Red	2	2	100	
Orange	3	3	1K	
Yellow	4	4	10K	
Green	5	5	100K	
Blue	6	6	1M	
Violet	7	7	10M	
Gray	8	8	100M	
White	9	9	1000M	

22.0 Speeds and Feeds

$N = \frac{CS\left(12\frac{in.}{ft}\right)}{\pi d}$ (22.1)

$f_m = f_t \cdot n_t \cdot N$ (22.2)

Plunge Rate = $\frac{1}{2} \cdot f_m$
N = spindle speed (rpm)
CS = cutting speed (ft/min)
d = diameter (in.)
f_m = feed rate (in./min)
f_t = feed (in./tooth/rev)
n_t = number of teeth

21.C Capacitor Code

Code	Tolerance
A	±0.05%
B	±0.1%
C	±0.25%
D	±0.5%
F	±1%
G	±2%
J	±5%
K	±10%
M or NONE	±20%
N	±30%
Q	−10%, +30%
S	−20%, +50%
T	−10%, +50%
Z	−20%, +80%

21.A Boolean Algebra

Boolean Theorems

$X \bullet 0 = 0$ (21.1)

$X \bullet 1 = X$ (21.2)

$X \bullet X = X$ (21.3)

$X \bullet \overline{X} = 0$ (21.4)

$X + 0 = X$ (21.5)

$X + 1 = 1$ (21.6)

$X + X = X$ (21.7)

$X + \overline{X} = 1$ (21.8)

$\overline{\overline{X}} = X$ (21.9)

Consensus Theorems

$X + \overline{X}Y = X + Y$ (21.16)

$X + \overline{X}\overline{Y} = X + \overline{Y}$ (21.17)

$\overline{X} + XY = \overline{X} + Y$ (21.18)

$\overline{X} + X\overline{Y} = \overline{X} + \overline{Y}$ (21.19)

DeMorgan's Theorems

$\overline{XY} = \overline{X} + \overline{Y}$ (21.20)

$\overline{X+Y} = \overline{X} \bullet \overline{Y}$ (21.21)

Commutative Law

$X \bullet Y = Y \bullet X$ (21.10)

$X + Y = Y + X$ (21.11)

Associative Law

$X(YZ) = (XY)Z$ (21.12)

$X + (Y + Z) = (X + Y) + Z$ (21.13)

Distributive Law

$X(Y+Z) = XY + XZ$ (21.14)

$(X+Y)(W+Z) = XW+XZ+YW+YZ$ (21.15)

23.0 G&M Codes

G00 = Rapid Traverse (23.1)
G01 = Straight Line Interpolation (23.2)
G02 = Circular Interpolation (clockwise) (23.3)
G03 = Circular Interpolation (c-clockwise) (23.4)
G04 = Dwell (wait) (23.5)
G05 = Pause for user intervention (23.6)
G20 = Inch programming units (23.7)
G21 = Millimeter programming units (23.8)
G80 = Canned cycle cancel (23.9)
G81 = Drilling cycle (23.10)
G82 = Drilling cycle with dwell (23.11)
G90 = Absolute Coordinates (23.12)
G91 = Relative Coordinates (23.13)
M00 = Pause (23.14)
M01 = Optional stop (23.15)
M02 = End of program (23.16)
M03 = Spindle on (23.17)
M05 = Spindle off (23.18)
M06 = Tool change (23.19)
M08 = Accessory # 1 on (23.20)
M09 = Accessory # 1 off (23.21)
M10 = Accessory # 2 on (23.22)
M11 = Accessory # 2 off (23.23)
M30 = Program end and reset (23.24)
M47 = Rewind (23.25)

24.0 Aerospace

Forces of Flight

$$C_D = \frac{2D}{A\rho v^2} \quad (24.1)$$

$$R_e = \frac{\rho v l}{\mu} \quad (24.2)$$

$$C_L = \frac{2L}{A\rho v^2} \quad (24.3)$$

$$M = Fd \quad (24.4)$$

C_L = coefficient of lift
C_D = coefficient of drag
L = lift
D = drag
A = wing area
ρ = density
R_e = Reynolds number
v = velocity
l = length of fluid travel
μ = fluid viscosity
F = force
m = mass
g = acceleration due to gravity
M = moment
d = moment arm (distance from datum perpendicular to F)

Bernoulli's Law

$$\left(P_s + \frac{\rho v^2}{2}\right)_1 = \left(P_s + \frac{\rho v^2}{2}\right)_2 \quad (23.16)$$

P_S = static pressure
v = velocity
ρ = density

Propulsion

$$F_N = W(v_j - v_o) \quad (24.5)$$

$$I = F_{ave}\Delta t \quad (24.6)$$

$$F_{net} = F_{avg} - F_g \quad (24.7)$$

$$a = \frac{v_f}{\Delta t} \quad (24.8)$$

F_N = net thrust
W = air mass flow
v_o = flight velocity
v_j = jet velocity
I = total impulse
F_{ave} = average thrust force
Δt = change in time (thrust duration)
F_{net} = net force
F_{avg} = average force
F_g = force of gravity
v_f = final velocity
a = acceleration
Δt = change in time (thrust duration)

NOTE: F_{ave} and F_{avg} are easily confused.

Atmosphere Parameters

$$T = 15.04 - 0.00649h \quad (24.17)$$

$$p = 101.29\left[\frac{(T + 273.1)}{288.08}\right]^{5.256} \quad (24.18)$$

$$\rho = \frac{p}{0.2869(T + 273.1)} \quad (24.19)$$

T = temperature
h = height
p = pressure
ρ = density

Energy

$$K = \tfrac{1}{2}mv^2 \quad (24.9)$$

$$U = \frac{-GMm}{R} \quad (24.10)$$

$$E = U + K = -\frac{GMm}{2R} \quad (24.11)$$

$$G = 6.67 \times 10^{-11}\frac{m^3}{kg \times s^2} \quad (24.12)$$

K = kinetic energy
m =mass
v = velocity
U = gravitational potential energy
G = universal gravitation constant
M =mass of central body
m = mass of orbiting object
R = Distance center main body to center of orbiting object
E = Total Energy of an orbit

Orbital Mechanics

$$e = \sqrt{1 - \frac{b^2}{a^2}} \quad (24.13)$$

$$T = 2\pi\frac{a^{\frac{3}{2}}}{\sqrt{\mu}} = 2\pi\frac{a^{\frac{3}{2}}}{\sqrt{GM}} \quad (24.14)$$

$$F = \frac{GMm}{r^2} \quad (24.15)$$

e = eccentricity
b = semi-minor axis
a =semi-major axis
T = orbital period
a = semi-major axis
μ = gravitational parameter
F = force of gravity between two bodies
G = universal gravitation constant
M =mass of central body
m = mass of orbiting object
r = distance between center of two objects

25.0 Environmental Sustainability

colonies/mL = # colonies/dilution (25.1)

$$\textbf{Transformation Efficiency} \ (\text{\# Transformants}/\mu g) = \frac{\text{\# of transformants}}{\mu g \text{ of DNA}} \cdot \frac{\text{final volume at recovery}}{\text{volume plated (mL)}} \quad (25.2)$$

$$\frac{\text{\# of moles of } CO_2}{\text{\# of moles of glucose produced in formula}} = \frac{\text{\# of moles consumed in experiment}}{\text{\# of moles of glucose produced in experiment}} \quad (25.3)$$

$$\textbf{Economic Growth} = \frac{GDP_2 - GDP_1}{GDP_1} \quad (25.4)$$

$$R_f = \frac{\text{distance the substance travels}}{\text{distance the solvent travels}} \quad (25.5)$$

GDP = gross domestic product
R_f = retention factor

26.0 USCS Soil Classification Chart