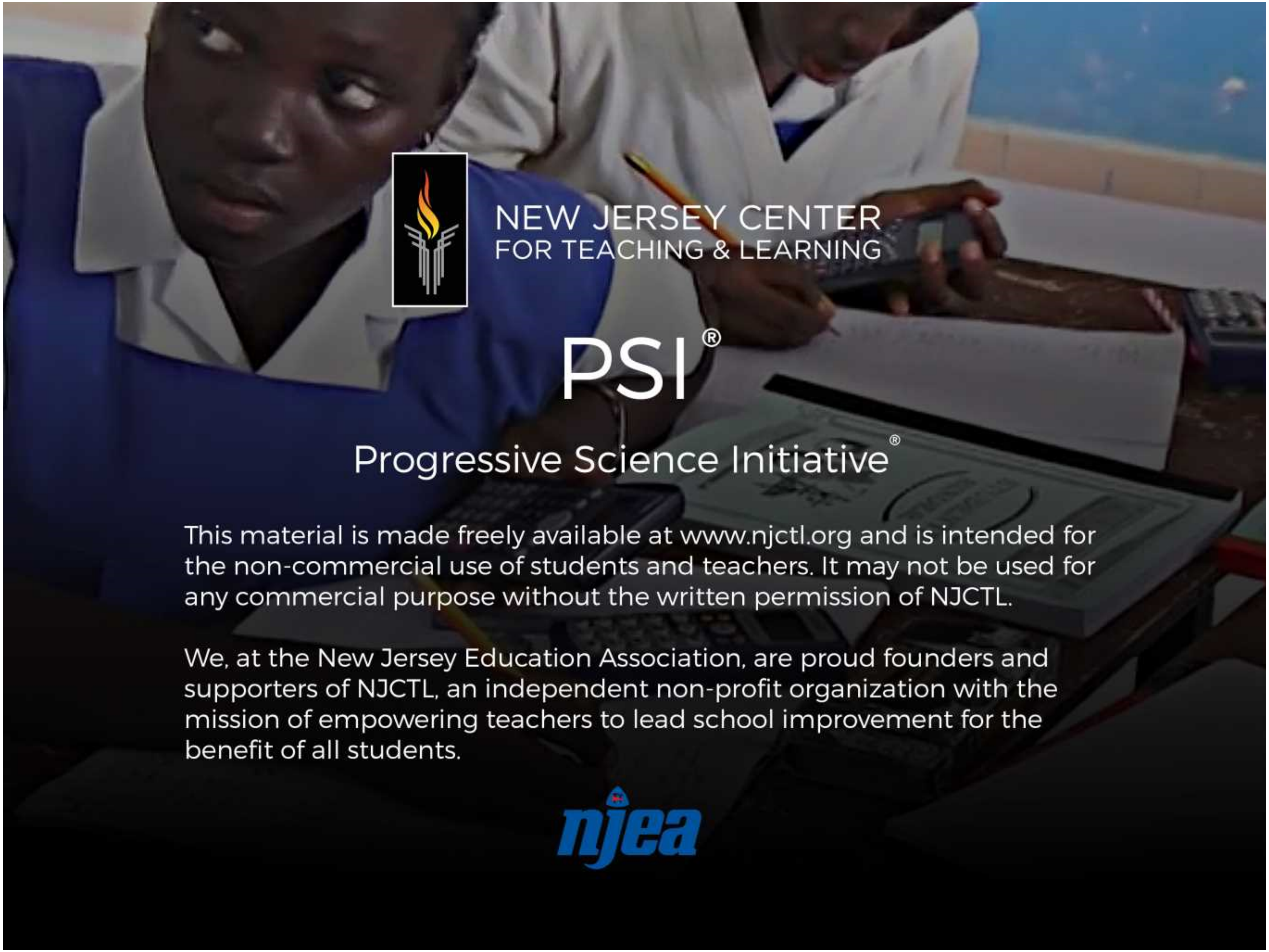

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AP Physics 1

Rotational Motion

2017-03-14

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Table of Contents

Click on the topic to go to that section

Axis of Rotation and Angular Properties

Return to Table of Contents

What is Rotational Motion?

Up until now, we have treated everything as if it were a either a point or a shape, but we would find its center of mass, and then pretend it acted like a point at its center of mass.

Continuing with our thread of starting simple and then layering on more reality, it is now time to address the fact that things also rotate - and many interesting applications arise from this.

What are some types of rotating objects and how are they important to society?

What is Rotational Motion?

The list is endless.

Here's some examples:

- Turbine generators that changes mechanical energy into electrical energy and supply electricity to businesses and homes.
- Helicopter blades that lift the helicopter into the air.
- Electric motors that power cars, circular saws and vacuum cleaners.
- Swivel chairs.
- Curve balls thrown by baseball pitchers.
- Automobile and truck tires that propel them down the road.

What is a common feature of these examples?

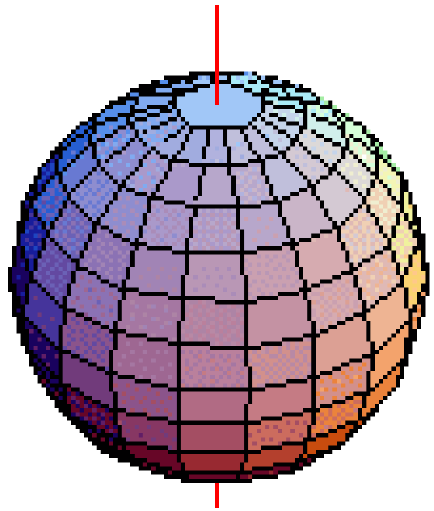
Axis of Rotation

They are all rotating about a line somewhere within the object called the axis of rotation.

We're also going to assume that all these objects are rigid bodies, that is, they keep their shape and are not deformed in any way by their motion.

Here's a sphere rotating about its axis of rotation - the vertical red line.

Does the axis of rotation have to be part of the rigid body?



Axis of Rotation

No - if you were to spin this donut around its center, the axis of rotation would be in the donut hole, pointing out of the page.

Angular Displacement

There are several theories why a circle has 360 degrees. Here's a few - if you're interested, there's a lot of information on the web.

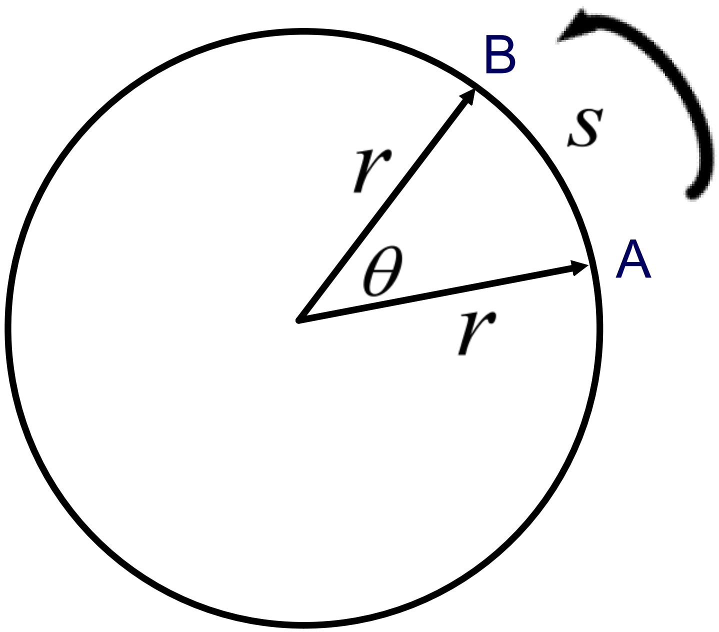
Some people believe it stems from the ancient Babylonians who had a number system based on 60 instead of our base 10 system.

Others track it back to the Persian or biblical Hebrew calendars of 12 months of 30 days each.

But - it has nothing to do with the actual geometry of the circle. There is a more natural unit - the radian.

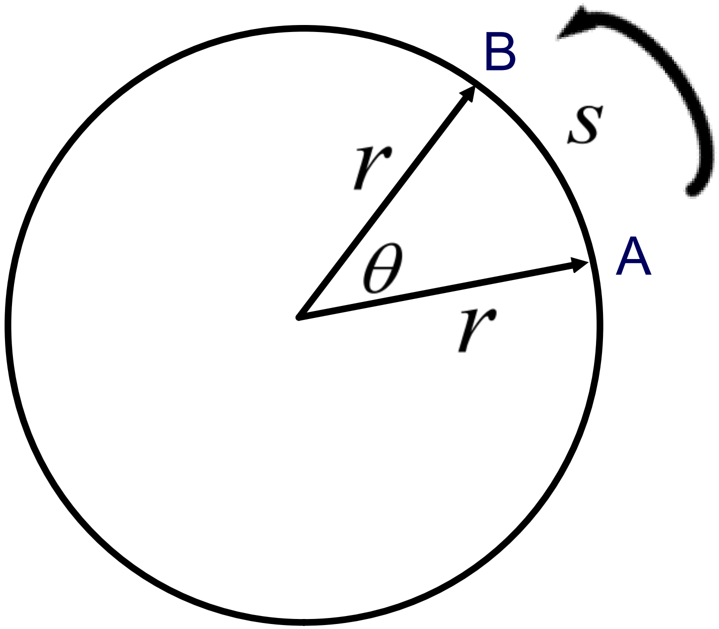
Angular Displacement

Let's look at this circle and assume it's rotating about its middle
- so the axis of rotation is pointing out of the board.



Start with a piece of the circle at point A. As the circle rotates counterclockwise, the piece of the circle reaches point B.

Angular Displacement

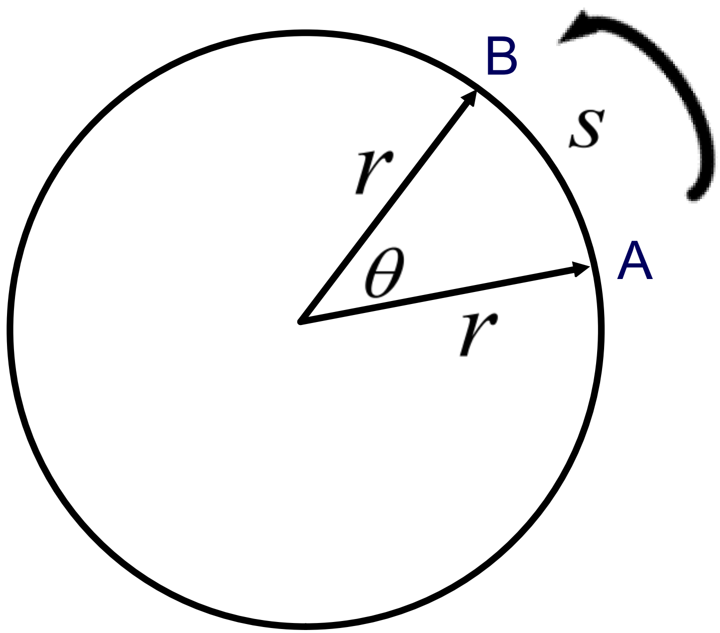


The point traveled a distance of s along the circumference, and swept out an angle θ. We can also say that the angle θ "subtends" an arc length of s.

Note that the points A and B are always at the same distance, r, from the axis of rotation. That's what it means to be a circle!

Angular Displacement

We will now define the angle of rotation, θ, as the ratio of the arc length, s, to the radius of the circle. Can you see how this is a much more natural definition of the angle of rotation then basing it on an old calendar or arbitrary numbering system?



$$\theta = \frac{s}{r}$$

We will call this angle of rotation, θ, the *angular displacement*.

Rotational angles are now defined in geometric terms - as the ratio of an arc length and the radius of the circle.

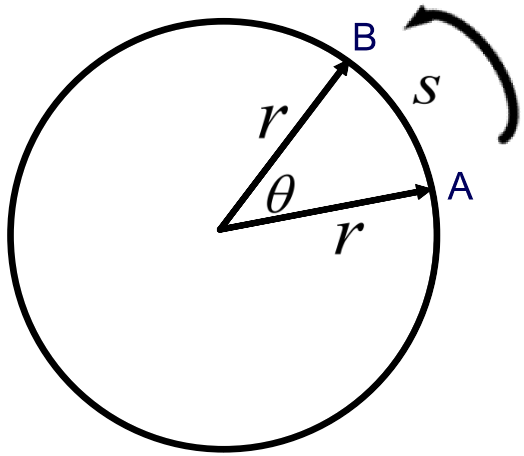
Radian

Angular displacement is *unit less* since it is the ratio of two distances. But, we will say that angular displacement is measured in *radians*. Let's relate this to concepts that we're pretty familiar with.

We know degrees, and we know that when a point on a circle rotates and comes back to the same point, it has performed one revolution - we start at point A, and rotate until we come back to point A.

What distance, s, was covered?

How many degrees were swept by this full rotation?



$$\theta = \frac{s}{r}$$

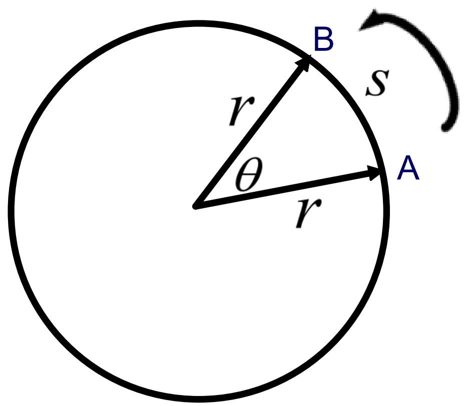
Radian

The point moved around the entire circumference, so it traveled $2\pi r$ while an angle of 360^0 was swept through. Using the angular displacement definition:

$$\theta = 360^0 = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi radians$$

$$360^0 = 2\pi radians$$

$$1\,radian \quad = \frac{360^0}{2\pi} = 57.3^0$$



$$\theta = \frac{s}{r}$$

Radian

When an object makes one complete revolution, it sweeps out an angle of 360^0 or 2π radians.

1 radian = 57.3^0

The radian is frequently abbreviated as rad.

You have to be careful about the settings on your calculator. Up until now, you probably just had angles set for degrees.

You need to set your calculator for radians. Please ask a classmate or your teacher for help on this.

1 What is the angle inside a circle, in radians, that subtends an arc length of 0.25 m? The radius of the circle is 5.0 m.

Answer

2 What is the value of $\pi/2$ radians in degrees?

A 0^0

B 45^0

C 90^0

D 180^0

Answer

3 What is the angular displacement for an arc length (s) that is equal to the radius of the circular rigid body?

A 0.5 rad

B 1.0 rad

C 0.5 π rad

D 1.0 π rad

Answer

4 A record spins 4 times around its center (4 revolutions). How many radians did it pass?

A π rad

B 2π rad

C 4π rad

D 8π rad

Answer

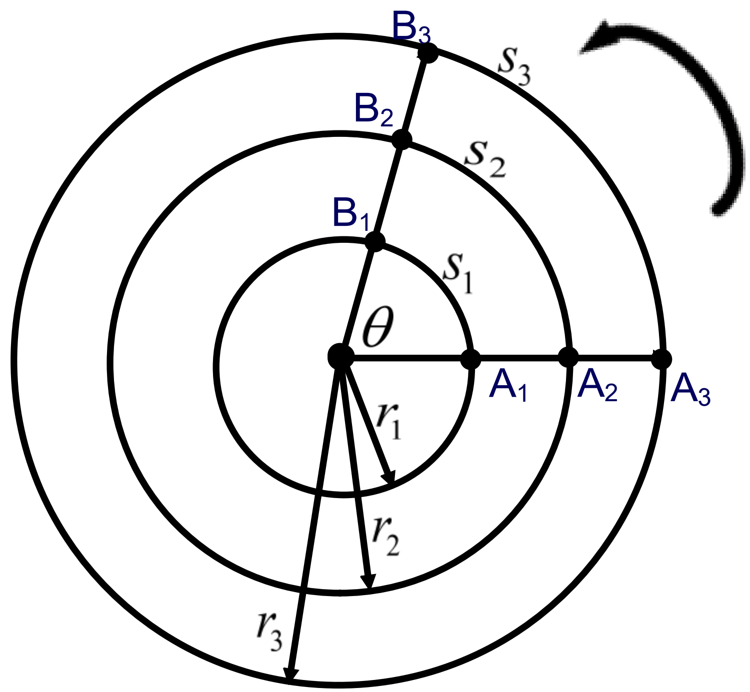
5 A circular hoop of radius 0.86 m rotates $\pi/3$ rad about its center. A small bug is on the hoop - what distance does it travel (arc length) during this rotation?

Answer

Angular Displacement

Something interesting - look at the three concentric circles drawn on the rigid disc, the radii r_i, arc lengths s_i, and the points A_i and B_i.

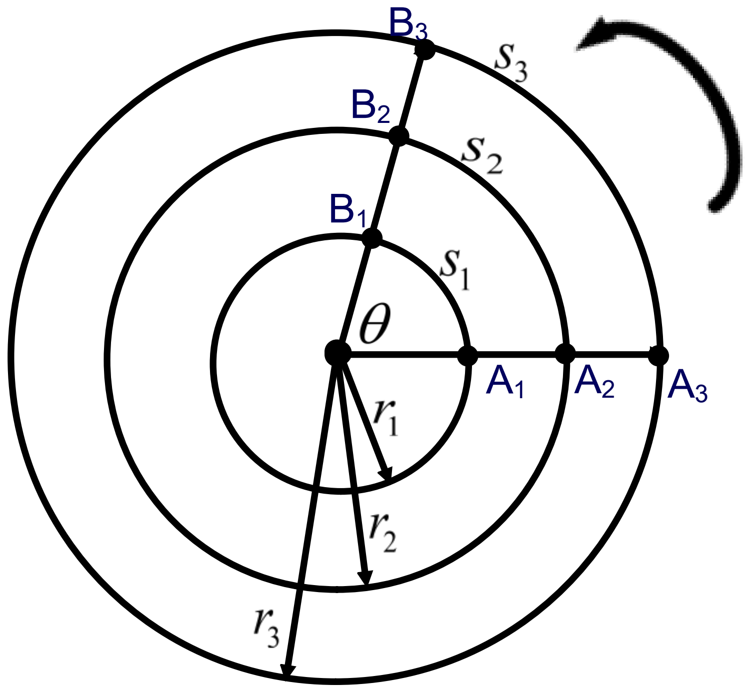
As the disk rotates, each point A_i moves to point B_i, covering the SAME angle θ, but covering a different arc length s_i.



Thus, all points on a rotating rigid disc move through the same angle, θ, but different arc lengths s_i, and different radii, r_i.

Angular Displacement

Since θ is constant for every point and describes something very fundamental about the rotating disc, it has been given a special name - angular displacement.

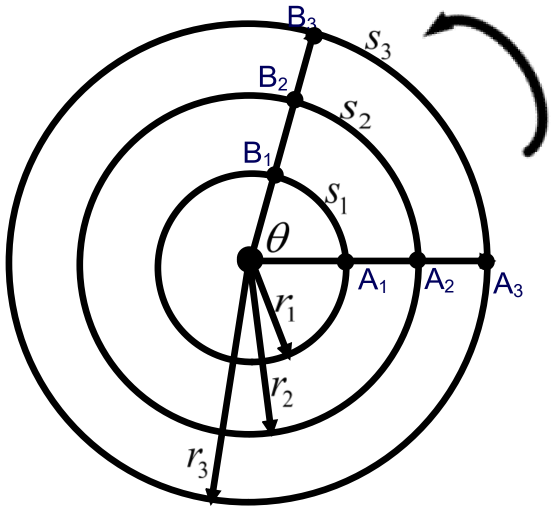


Angular Displacement

We're now in a position to relate angular motion to linear motion. Linear motion has been covered in the Kinematics section of this course.

The radian is now defined. Angular Displacement is defined.

An object at point A_1 rotates to point B_1, covering a linear displacement of s_1 and an angular displacement of θ; similarly for points A_2 and A_3.



Angular Displacement

Using the definition of the radian for these three points we show how the linear displacement any point on the disc increases directly as the distance of the point from the axis of rotation (its radius).

The angular displacement, θ, remains the same for a constant rotation for all points on the disc.

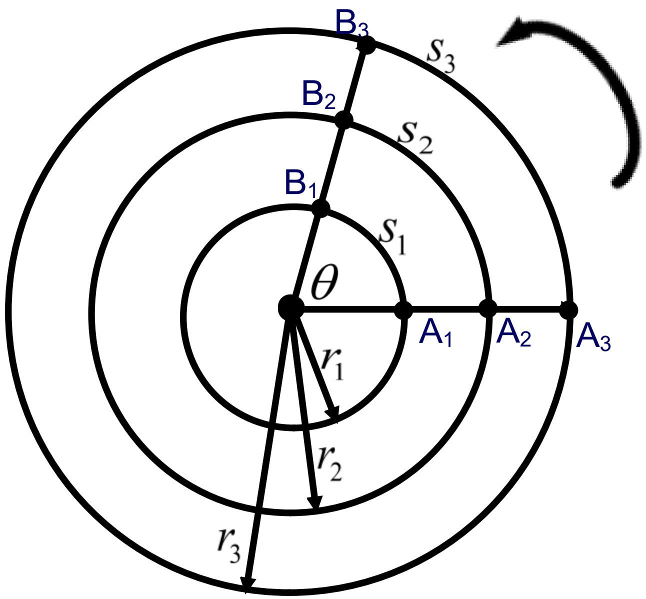
$$\theta = \frac{s}{r}$$

$$s = r\theta$$

$$s_1 = r_1\theta$$

$$s_2 = r_2\theta$$

$$s_3 = r_3\theta$$



6 The following are properties of angular displacement, θ, and linear displacement, s. Select two answers.

A θ, for points on a rotating object, depends on their distance from the axis of rotation.

B θ, for points on a rotating object, does not depend on their distance from the axis of rotation.

C The linear displacement, for points on a rotating object, depends on their distance from the axis of roation.

D The linear displacement, for points on a rotating object, does not depend on their distance from the axis of roation.

Answer

7 There are two people on a merry go round. Person A is 2.3 m from the axis of rotation. Person B is 3.4 m from the axis of rotation. The merry go round moves through an angular displacement of $\pi/4$. What linear displacement (arc length) is covered by both people? Compare and contrast these motions.

Answer

Other Angular Quantities

Angular displacement is now related to Linear displacement.

We also spent time earlier in this course working on Kinematics problems with linear displacement.

Let's continue with angular motion. What other quantities played a key role in linear motion?

Velocity and Acceleration which were defined as:

$$v = \frac{\Delta x}{\Delta t} \qquad a = \frac{\Delta v}{\Delta t}$$

Other Angular Quantities

Define Angular Velocity as the change in angular displacement over time.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Define Angular Acceleration as the change in angular velocity over time.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Note *The Greek letter equivalents of "v" and "a" were taken for the angular equivalents of the linear motion variables for our notation.*

Other Angular Quantities

Since ω and α are both related to θ, and θ is the same for all points on a rotating rigid body, ω and α are also the same for all points on a rotating rigid body.

But the same is not true for linear velocity and acceleration.

Let's relate the linear and angular velocity and acceleration to their angular equivalents.

$$\omega = \frac{\Delta\theta}{\Delta t} \qquad \alpha = \frac{\Delta\omega}{\Delta t}$$

Angular and Linear Velocity

Start with angular velocity, and substitute in the linear displacement for the angular displacement.

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta\left(\frac{s}{r}\right)}{\Delta t}$$

r is constant, so move it outside the Δ

$$\omega = \frac{1}{r}\frac{\Delta s}{\Delta t} = \frac{1}{r}v$$

$$v = r\omega$$

This confirms what you feel on a merry go round - the further away from the center you move, the faster you feel you're going - the linear velocity!

Angular and Linear Acceleration

Start with angular acceleration, and substitute in the linear velocity for the angular velocity.

Please work with your group and derive the relationship between angular and linear acceleration. Then move the screen below to check your work.

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\Delta\left(\frac{v}{r}\right)}{\Delta t}$$

r is constant, so move it outside the Δ

$$\alpha = \frac{1}{r}\frac{\Delta v}{\Delta t} = \frac{1}{r}a$$

$$a = r\alpha$$

Angular and Linear Quantities Summary

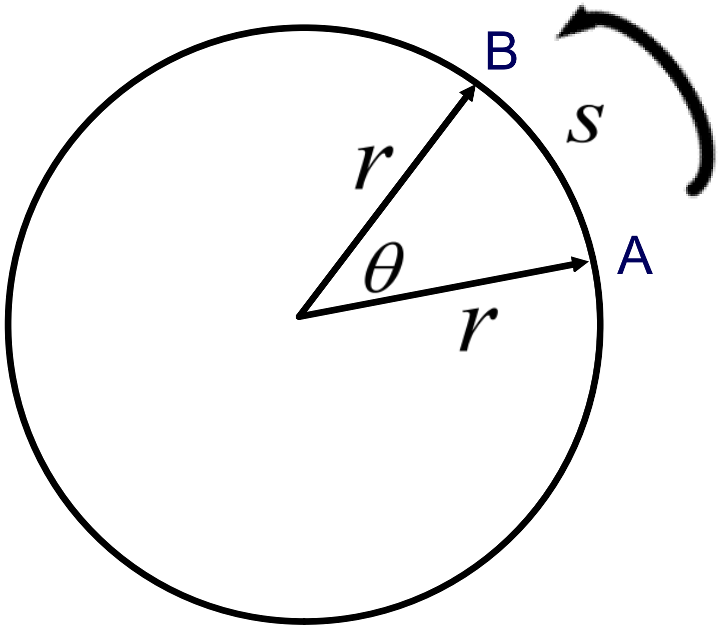
	Angular	Linear	Relationship
Displacement	θ	s	$s = r\theta$
Velocity	$\omega = \dfrac{\Delta\theta}{\Delta t}$	$v = \dfrac{\Delta s}{\Delta t}$	$v = r\omega$
Acceleration	$\alpha = \dfrac{\Delta\omega}{\Delta t}$	$a = \dfrac{\Delta v}{\Delta t}$	$a = r\alpha$

Angular Velocity sign

We're familiar with how to assign positive and negative values to displacement. Typically if you move to the right or up, we give that a positive displacement (of course, this is arbitrary, but it's a pretty standard convention).

How do we assign a "sign" for angular displacement, velocity or acceleration?

Does "right" or "up" work with rotational motion? Does it work with a circle?



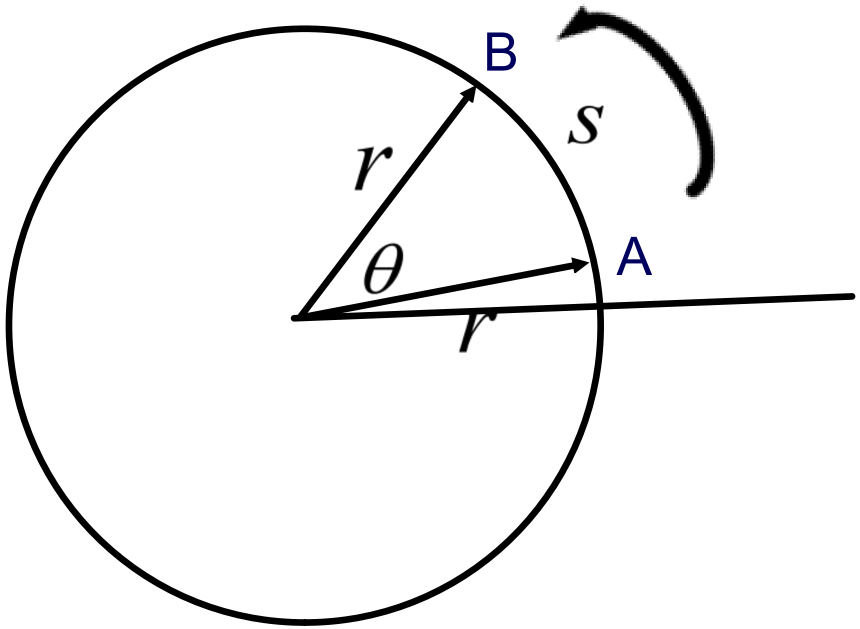
Angular Velocity sign

Not really. Not at all.

From your math classes, you know that the horizontal axis through a circle is labeled as 0^0, and angles are measured in a counter clockwise direction.

Once we agree on that, let's look at the definition of angular velocity:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

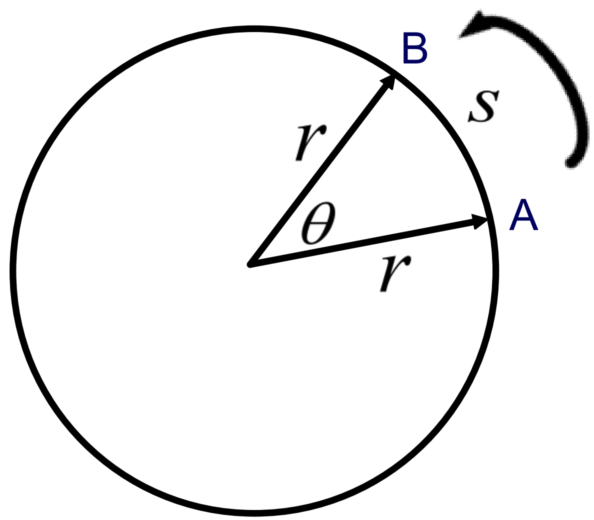


Angular Velocity sign

As the disc rotates in a counter clockwise fashion, θ increases, so Δθ is positive. Since Δt is also positive, Δθ/Δt must be positive.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Thus, Counter clockwise rotations result in a positive ω.



8 Explain why a disc that rotates clockwise leads to a negative value for angular velocity, ω. You can use an example to show your point.

Answer

9 An object moves $\pi/2$ radians in 8 s. What is its angular velocity?

Answer

10What is the linear speed of a child on a merry-go-round of radius 3.0 m that has an angular velocity of 4.0 rad/s?

A 12 m/s

B 0.75 m/s

C 1.3 m/s

D 10 m/s

Answer

11 What is the angular velocity of an object traveling in a circle of radius 0.75 m with a linear speed of 3.5 m/s?

Answer

Angular Acceleration sign

Start with the definition of angular acceleration:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

There are four cases here, all similar to the case of linear acceleration. Can you figure them out?

Here's a hint - for linear acceleration, if an object's velocity is increasing in the same direction of its displacement, the acceleration is positive.

If the object's velocity is decreasing in the direction of its displacement, then its acceleration is negative.

Angular Acceleration sign

Here they are:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

ω increases in counter clockwise direction - α is positive
ω decreases in the counter clockwise direction - α is negative

ω increases in clockwise direction - α is negative
ω decreases in the clockwise direction - α is positive

12 What is the angular acceleration of a ball that starts at rest and increases its angular velocity uniformly to 5 rad/s in 10 s?

Answer

13 What is the angular velocity of a ball that starts at rest and rolls for 5 s with a constant angular acceleration of 20 rad/s^2?

A 10 rad/s

B 4 rad/s

C 7 rad/s

D 100 rad/s

Answer

Three types of Acceleration

So, we have two types of displacement, velocity and acceleration - linear and angular.

However, this picture is not quite complete.

We learned about a third type of acceleration earlier.

What is it?

Three types of Acceleration

Centripetal! This was covered earlier in the Uniform Circular Motion chapter - and you can go back to there to review if you'd like.

This is the acceleration that a point on a rotating disc feels that keeps it moving in a circular direction.

$$a_c = \frac{v^2}{r}$$

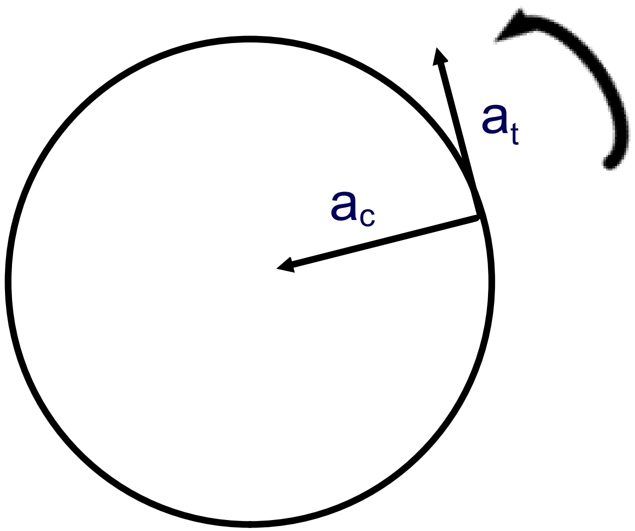
The v in the above equation is the v we've been working with, which is the tangential velocity of an object located on a rotating object.

Up until now, we've just called it the linear velocity - but we're going to need its tangential nature to discuss acceleration.

Three types of Acceleration

Look at the disc below which shows the linear acceleration that we've defined as well as the centripetal acceleration.

But, centripetal acceleration is also linear! So we will rename the linear acceleration we've been dealing with in this chapter as tangential acceleration - as it is tangent to the rotating object.



The angular acceleration, α, is a vector that points out of the page. Don't worry about that now - vectors pointing out of the page will be covered in the Electricity and Magnetism section of this course - and in AP C Mechanics.

Three types of Acceleration

We now have three accelerations associated with a rigid rotating body: tangential, angular and centripetal, or a_t, α, and a_C.

And they are all related to each other - can you express all three of these accelerations in terms of the angular velocity of the rotating object? Give it a try with these equations as a start:

$$a_t = r\alpha$$

$$a_c = \frac{v^2}{r}$$

$$v = r\omega$$

Look over here for the answer →

$$a_t = r\alpha = r\frac{\Delta\omega}{\Delta t}$$

$$a_c = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

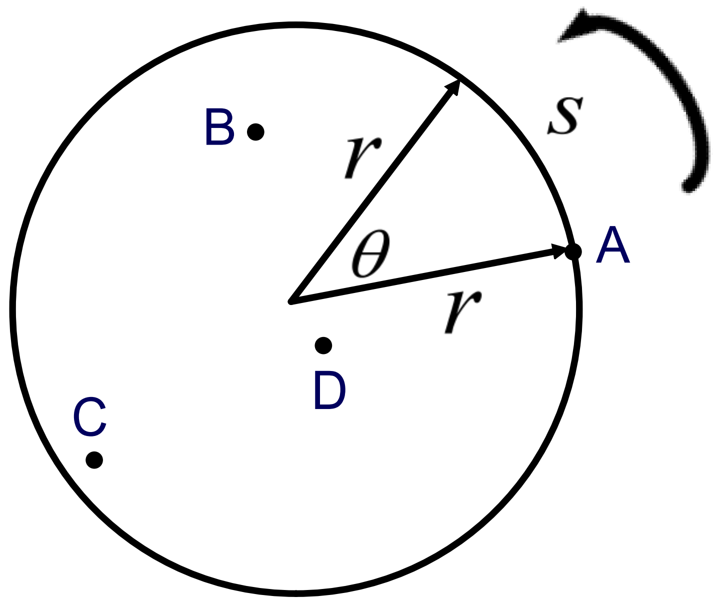
14 At which point is the magnitude of the centripetal acceleration the greatest?

A A

B B

C C

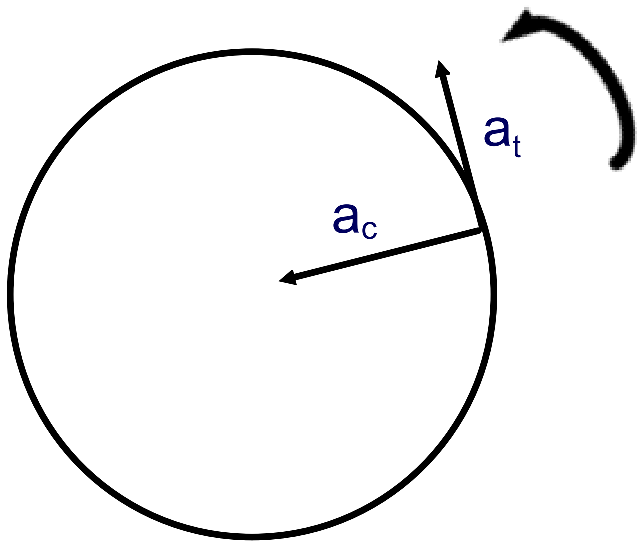
D D



Answer

Total Linear Acceleration

There are two flavors of linear acceleration - tangential and centripetal. They are perpendicular to each other, so what mathematical equation would you use to sum them up for a total linear acceleration?

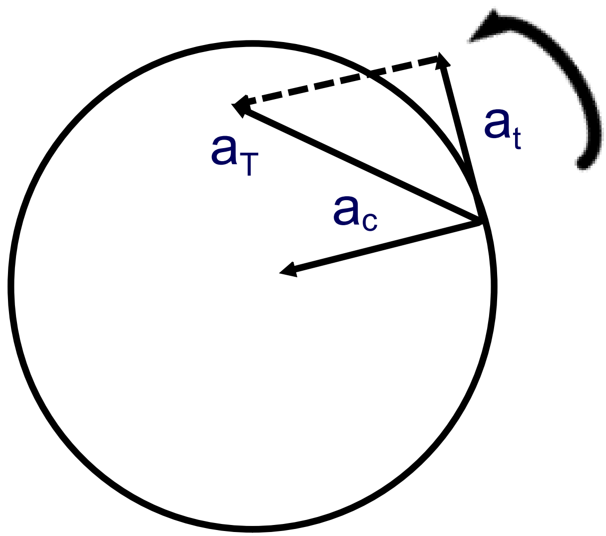


Total Linear Acceleration

You take the vector sum: $\vec{a}_{Total} = \vec{a}_T = \vec{a}_t + \vec{a}_c$

and use the Pythagorean Theorem to find the magnitude of a_{total}.

$$a_T = \sqrt{a_t^2 + a_c^2}$$



15 A child pushes, with a constant force, a merry-go-round with a radius of 2.5 m from rest to an angular velocity of 3.0 rad/s in 8.0 s. What is the merry-go-round's tangential acceleration?

A $0.25\ m/s^2$

B $0.35\ m/s^2$

C $0.62\ m/s^2$

D $0.94\ m/s^2$

Answer

16 A child pushes, with a constant force, a merry-go-round with a radius of 2.5 m from rest to an angular velocity of 3.0 rad/s in 8.0 s. What is the merry-go-round's centripetal acceleration at t = 8.0s?

A 7.5 m/s^2

B 19 m/s^2

C 23 m/s^2

D 46 m/s^2

Answer

17 A child pushes, with a constant force, a merry-go-round with a radius of 2.5 m from rest to an angular velocity of 3.0 rad/s in 8.0 s. What is the merry-go-round's total linear acceleration at t = 8.0s?

Answer

18 A bear pushes, with a constant force, a circular rock with a radius of 7.2 m from rest. The rock rotates with a centripetal acceleration of 4.0 m/s^2 at t = 5.0 s. What is its angular velocity at t = 5.0 s?

A 0.65 rad/s

B 0.75 rad/s

C 0.25 rad/s

D 0.33 rad/s

Answer

19 A bear pushes, with a constant force, a circular rock with a radius of 7.2 m from rest. The rock rotates with a centripetal acceleration of 4.0 m/s^2 at $t = 5.0$ s. What is its tangential acceleration at $t = 5.0$ s?

Answer

20 A bear pushes, with a constant force, a circular rock with a radius of 7.2 m from rest. The rock rotates with a centripetal acceleration of 4.0 m/s^2 and a tangential acceleration of 1.1 m/s^2 at t = 5.0 s. What is its total linear acceleration at that time?

A 1.3 m/s^2

B 1.4 m/s^2

C 3.1 m/s^2

D 4.1 m/s^2

Answer

Angular Velocity and Frequency

Frequency, f, is defined as the number of revolutions an object makes per second and f = 1/T, where T is the period.

A rotating object moves through an angular displacement, θ = 2πr radians in one revolution - 1 period.

The linear velocity of any point on a rotating object is

$$v = \frac{\Delta s}{\Delta t} = \frac{2\pi r}{T} = 2\pi r f$$

This is very useful for solving rotational motion problems.

Try putting these facts together to derive a relationship between angular velocity and frequency.

Angular Velocity and Frequency

$$v = \frac{\Delta s}{\Delta t} = \frac{2\pi r}{T} = 2\pi r \frac{1}{T} = 2\pi r f$$

$$f = \frac{v}{2\pi r} = \frac{r\omega}{2\pi r} = \frac{\omega}{2\pi}$$

$$\omega = 2\pi f$$

The angular velocity of a rotating object is equal to 2π times its frequency of rotation.

21 A tire with a radius of 4.0 m rolls with an angular velocity of 8.0 rad/s. What is the frequency of the tire's revolutions? What is its period?

Answer

22 A ball with a radius of 2.0 m rolls with a velocity of 3.0 m/s. What is the frequency of the ball's revolutions? What is its period?

A 0.24 rev/s 4.2 s

B 0.24 rev/s 8.4 s

C 0.48 rev/s 2.1 s

D 0.48 rev/s 4.2 s

Answer

23 Four different objects rotate with the following parameters. In which cases are the frequency of the objects's revolutions identical? Select two answers.

A $v = 4$ m/s $r = 1$ m

B $v = 8$ m/s $r = 2$ m

C $v = 1$ m/s $r = 1$ m

D $v = 3$ m/s $r = 6$ m

Answer

Rotational Kinematics

Return to Table
of Contents

Translational Kinematics

Now that all the definitions for rotational quantities have been covered, we're almost ready to discuss the motion of rotating rigid objects - Rotational Kinematics.

First, let's review the equations for translational kinematics - this was one of the first topics that was covered in this course.

These equations are ONLY valid for cases involving a constant acceleration.

This allows gravitational problems to be solved (a = g, a constant) and makes it possible to solve kinematics equations without advanced calculus. When acceleration changes, the problems are more complex.

Translational Kinematics

Here are the equations. If you would like, please review their derivations in the Kinematics section of this course.

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$\overline{v} = \frac{v + v_0}{2}$$

Rotational Kinematics

For each of the translational kinematics equations, there is an equivalent rotational equation. We'll derive one of them and just present the others - basically you just replace each translational variable with its rotational analog.

$$v = v_0 + at$$

$$r\omega = r\omega_0 + r\alpha t$$

$$\omega = \omega_0 + \alpha t$$

Rotational Kinematics

Here they are. We've been using Δs for linear displacement, but Δx is just fine. Similarly to the translational equations, these only work for constant angular acceleration.

$$\omega = \omega_0 + \alpha t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$\overline{\omega} = \frac{\omega + \omega_0}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$\overline{v} = \frac{v + v_0}{2}$$

Rolling Motion

The rotational motion equations also work as part of the equations for rolling motion.

Rolling motion is simply rotational motion and linear motion combined.

Think of a wheel on a car:

It is rotating, showing rotational motion.

It also moves forward as it rotates, therefore showing linear motion.

Thus, we use the translational and rotational kinematics equations to solve rolling motion problems.

24 A bicycle wheel with a radius of 0.30 m starts from rest and accelerates at a rate of 4.5 rad/s^2 for 11 s. What is its final angular velocity?

Answer

25 A bicycle wheel with a radius of 0.30 m starts from rest and accelerates at a rate of 4.5 rad/s^2 for 11 s. What is its final linear velocity?

A 20 m/s

B 15 m/s

C 10 m/s

D 5 m/s

Answer

26 A bicycle wheel with a radius of 0.300 m starts from rest and accelerates at a rate of 4.50 rad/s^2 for 11.0 s. What is its angular displacement during that time?

Answer

27 A bicycle wheel with a radius of 0.30 m starts from rest and accelerates at a rate of 4.5 rad/s^2 for 11 s. How many revolutions did it make during that time?

Answer

28 A bicycle wheel with a radius of 0.30 m starts from rest and accelerates at a rate of 4.5 rad/s^2 for 11 s. What is its linear displacement during that time?

A 40 m

B 82 m

C 160 m

D 210 m

Answer

29 A 50.0 cm diameter wheel accelerates from 5.0 revolutions per second to 7.0 revolutions per second in 8.0 s. What is its angular acceleration?

A π rad/s^2

B $\pi/2$ rad/s^2

C $\pi/3$ rad/s^2

D $\pi/6$ rad/s^2

Answer

30 A 50.0 cm diameter wheel accelerates from 5.0 revolutions per second to 7.0 revolutions per second in 8.0 s. What is its angular displacement during that time?

Answer

31 A 50.0 cm diameter wheel accelerates uniformly from 5.0 revolutions per second to 7.0 revolutions per second in 8.0 seconds. What linear displacement, s, will a point on the outside edge of the wheel have traveled during that time?

Answer

Rotational Dynamics

Return to Table
of Contents

Rotational Dynamics

Just like there are rotational analogs for Kinematics, there are rotational analogs for Dynamics.

Kinematics allowed us to solve for the motion of objects, without caring why or how they moved.

Dynamics showed how the application of forces causes motion - and this is summed up in Newton's Three Laws.

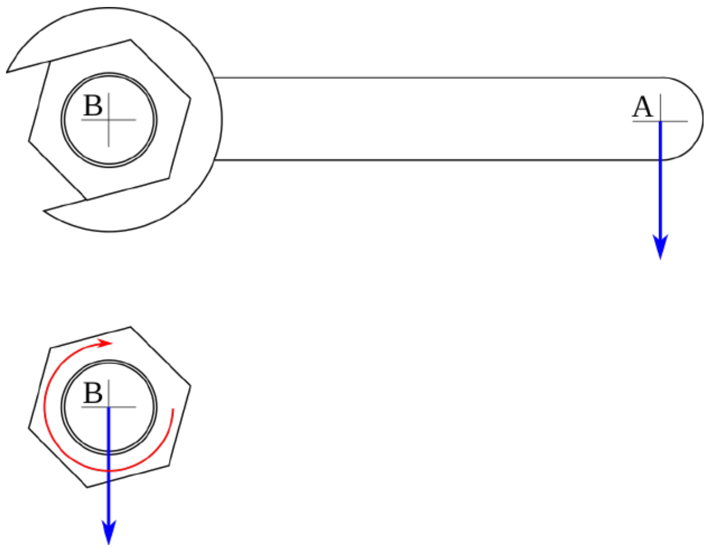
These laws also apply to rotational motion, but will be in a slightly different format.

Torque

Forces underlie translational dynamics. A new term, related to force, is the foundation of rotational motion. It is called Torque.

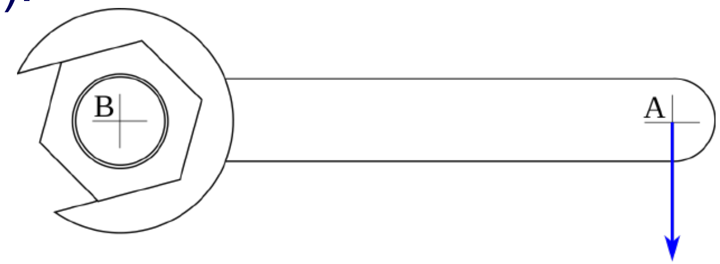
The wrench is about to turn the nut by a force applied at point A. Will the nut turn, and in which direction?

Consider the axis of rotation of the nut to be through its center.

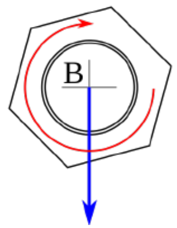


Torque

The nut will turn in a clockwise direction - recall that "left" and "right" have no meaning in rotational motion. This is good - a force causes motion from rest which implies an acceleration (but its an angular acceleration, so it's a little new).



What if a force was applied to the nut at point B? The nut is attached to a bolt that is stuck in the wall. Will the bolt rotate or move?

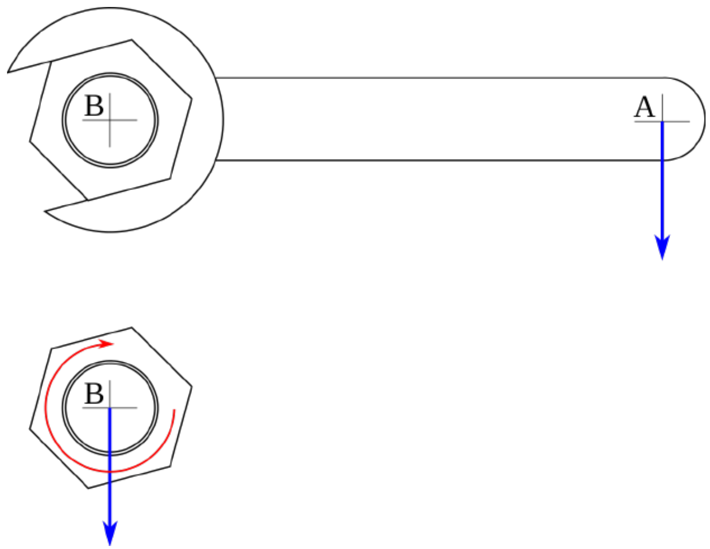


http://commons.wikimedia.org/wiki/File%3AAction_meca_equivalence_force_couple_plan.svg

Torque

No, it will not rotate or move at all. It makes sense that the nut won't move translationally as the friction of the bolt and the wood holding it is providing an equal and opposite force. But, it did rotate when the force was applied at point A.

Assuming the same magnitude force was applied at points A and B, what is the difference between the two cases? Why is it not rotating now?

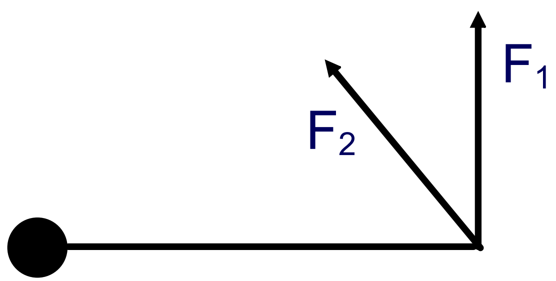


Torque

The distance between where the force was applied and the axis of rotation. When the force was applied at the axis of rotation, the nut did not move.

There's just one more variable to consider for torque. Here's a picture of a door (the horizontal line) and its door hinge (the dot).

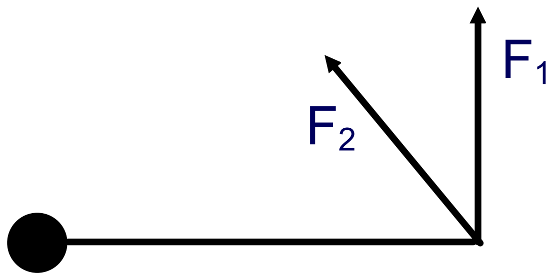
Let's say you want to open the door by pushing it - rotating it about the hinge. The magnitude of F_1 and F_2 is the same.



Which force will cause the door to open quicker (have a greater angular acceleration)?

Torque

You've probably known this since you first started walking - F_1 gives the greater angular acceleration. Pushing perpendicular to a line connecting the application of the force to the axis of rotation results in the greater angular acceleration.



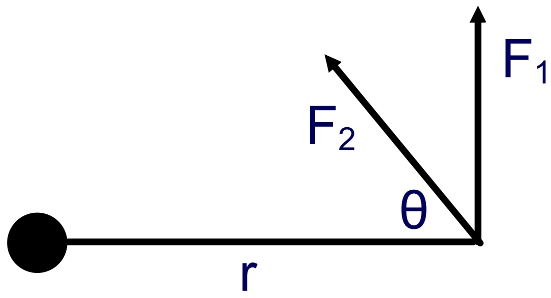
It will also allow a smaller force to have the same impact as a greater force applied at an angle different than 90^0.

Torque

This combination of Force applied at a distance and an angle from the axis of rotation that causes the angular acceleration is called Torque, which is represented by the Greek letter tau:

$$\tau = rF\sin\theta$$

Torque is a vector - the above equation shows the magnitude of the torque.

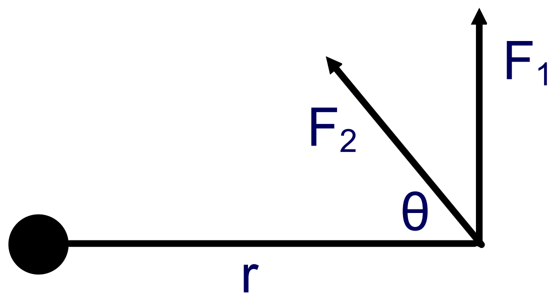


If the object rotates in a counter clockwise, the torque is positive. Negative values of torque cause clockwise rotations.

Torque

The sin θ takes into account that the closer the force is applied perpendicular to the line connected to the axis of rotation results in a greater torque - a greater angular acceleration. θ is the angle between the applied force and the line connecting it to the axis of rotation.

$$\tau = rF\sin\theta$$



When you get to vector calculus, the equation becomes more elegant involving the cross product:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Torque

The units of torque are Newton-meters.

$$\tau = rF\sin\theta$$

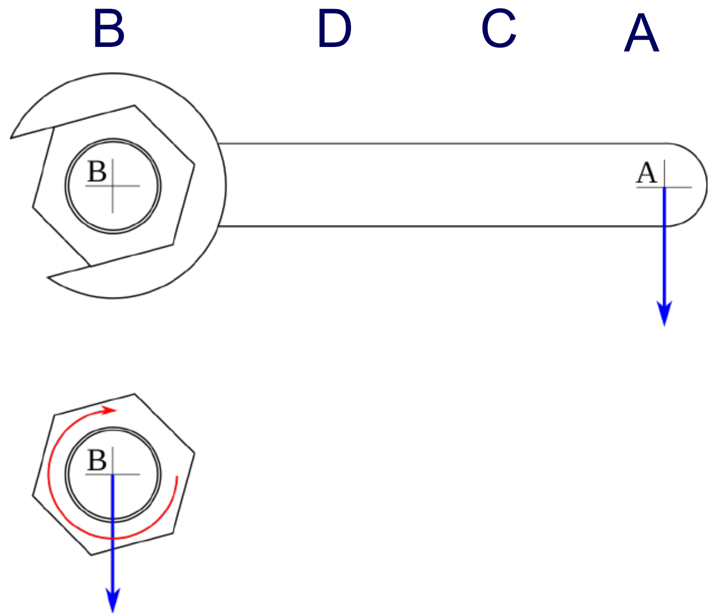
Note how this is similar to work and energy - and in those cases, we replace N-m with Joules.

Torque, however is a vector.

How is this different from work and energy?

Work and energy are scalars. So, to emphasize this difference, torque is never expressed in terms of Joules.

32 Assume a force is applied perpendicular to the line connecting it to the axis of rotation of the wrench-nut system. At which point will the nut experience the greatest torque?



A

B

C

D

http://commons.wikimedia.org/wiki/File%3AAction_meca_equivalence_force_couple_plan.svg

Answer

33 A force of 300 N is applied to a crowbar 20 cm from the axis of rotation, and perpendicular to the line connecting it to the axis of rotation. Calculate the magnitude of the torque.

A 50 N-m

B 60 N-m

C 70 N-m

D 80 N-m

Answer

34 A force of 300 N is applied to a crow bar 20 cm from the axis of rotation at an angle of 25^0 to a line connecting it to the axis of rotation. Calculate the magnitude of the torque.

A 30 N-m

B 25 N-m

C 20 N-m

D 15 N-m

Answer

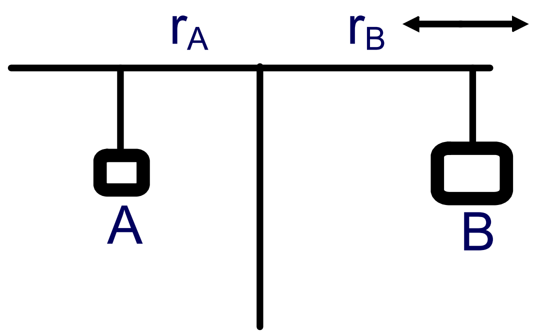
Torque Applications

Torque will be illustrated by three examples:

- Balancing masses
- Lever
- Screwdriver handle

Balance

Here is a balance with two masses attached to it. Block A has a mass of 0.67 kg and is located 0.23 m from the pivot point.



Block B has a mass of 1.2 kg. Where should it be located so that the balance beam is perfectly horizontal and is not moving?

Is this a translational motion or a rotational motion problem?

See-Saw (balance)

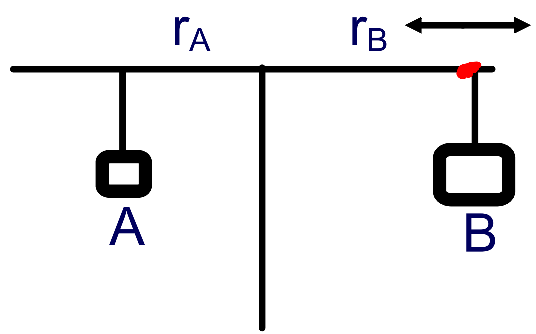
Here's an artistic hint. The two boys on the left represent mass A and the two slightly larger boys on the right represent mass B.

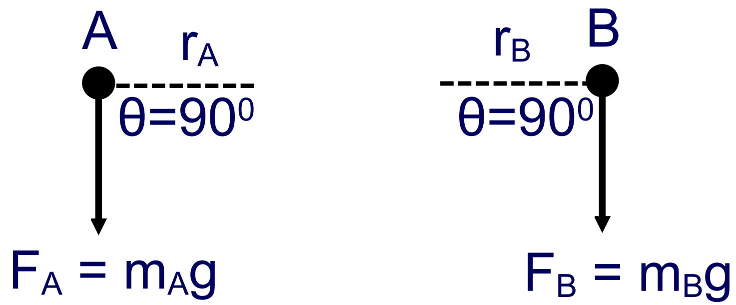
Winslow Homer, "The See-Saw" 1873

A side note - there are many connections between physics and art - this would be a good project to research.

Balance

As with all Dynamics problem, let's start with free body diagrams - but we'll take the points on the balance beam that the mass's are attached to:





$$\tau_A = r_A F_A \sin\theta \qquad \tau_B = -r_B F_B \sin\theta$$

Note the signs - mass A is trying to rotate the beam counterclockwise - so its torque is positive. Mass B is trying to rotate the beam clockwise - negative torque; we have a rotational problem.

Balance

This is a rotational motion problem, so in order to have the beam not rotate, the sum of the torques applied to it must equal zero.

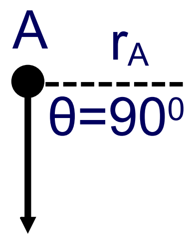
$$\Sigma\tau = 0$$

$$r_A F_A \sin\theta - r_B F_B \sin\theta = 0$$

$$r_A m_A g - r_B m_B g = 0$$

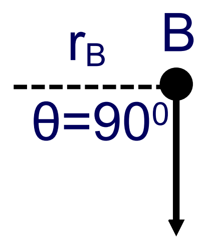
$$r_B = \frac{m_A}{m_B} r_A$$

$$r_B = \frac{0.67kg}{1.2kg}(0.23m) = 0.13m$$



$F_A = m_A g$

$$\tau_A = r_A F_A \sin\theta$$

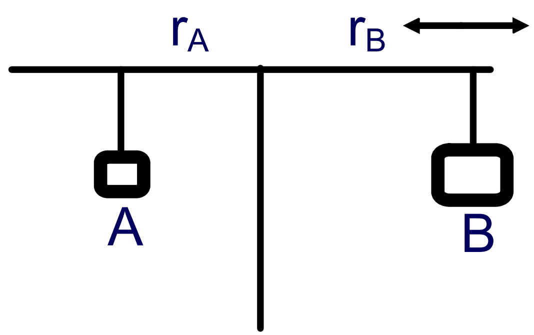


$F_B = m_B g$

$$\tau_B = -r_B F_B \sin\theta$$

Balance and See-Saw

Again, many elementary school students already know this! If you're on a see-saw and you have to balance several students on the other side, you position yourself further from the fulcrum than they are.



Winslow Homer, "The See-Saw" 1873

35 John and Sally are sitting on the opposite sides of a See-Saw, both of them 5.0 m distant from the fulcrum. Sally has a mass of 45 kg. What is the gravitational force on John if the See-Saw is not moving? Use $g = 10\ m/s^2$.

A 450 N

B 400 N

C 350 N

D 300 N

Answer

36 A student wants to balance two masses of 2.00 kg and 0.500 kg on a meter stick. The 0.500 kg mass is 0.500 m from the fulcrum in the middle of the meter stick. How far away should she put the 2.00 kg mass from the fulcrum?

A 0.500 m

B 0.333 m

C 0.250 m

D 0.125 m

Answer

37 Sally wants to lift a 105 kg rock off the ground with a 3.00 m bar. She positions the bar on a fulcrum (a tree log 1.00 m away from the rock) and wedges it under the rock, at an angle of 45^0 with the ground. She exerts a force of 612 N perpendicular to the ground. How far away from the fulcrum should the force be applied to move the rock? Use $g = 10\ m/s^2$.

Answer

38 Sally wants to lift a 105 kg rock off the ground with a 3.00 m bar. She positions the bar on a fulcrum (a tree log 1.00 m away from the rock) and wedges it under the rock, at an angle of 45^0 with the ground. She pushes on the bar at a distance of 1.8 m away from the fulcrum and starts moving the rock. Joe then comes by, and he wants to try moving the rock, but he is not as strong as Sally. What should he do? Select two answers.

A Push on the bar further from the fulcrum.

B Push on the bar closer to the fulcrum.

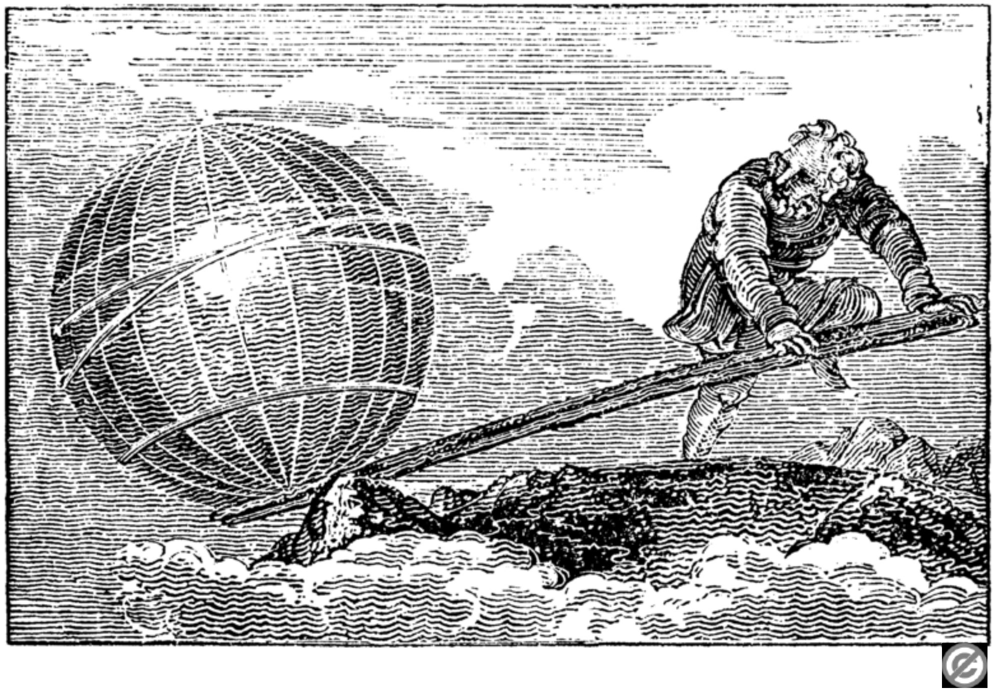
C Move the fulcrum closer to the rock.

D Move the fulcrum further away from the rock.

Answer

Lever

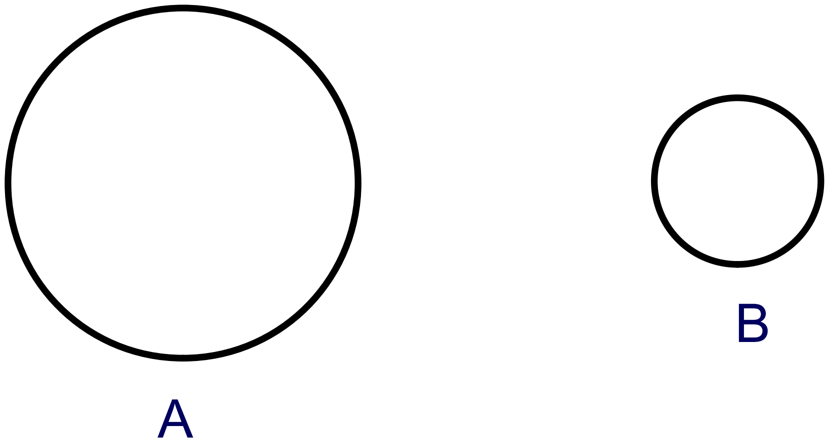
A remark attributed to Archimedes, "Give me a place to stand and with a lever I will move the whole world," also applies to torque. See how Archimedes has set the earth up so it is closer to the fulcrum, reducing the amount of force he needed to exert to move it (of course, this is a fanciful illustration).



Screwdriver handle

Here's a view of a screwdriver from the top - we're looking at the handle, and the shank (that's the long thin part that connects the handle to the tip which contacts the screw) of the screwdriver goes into the page.

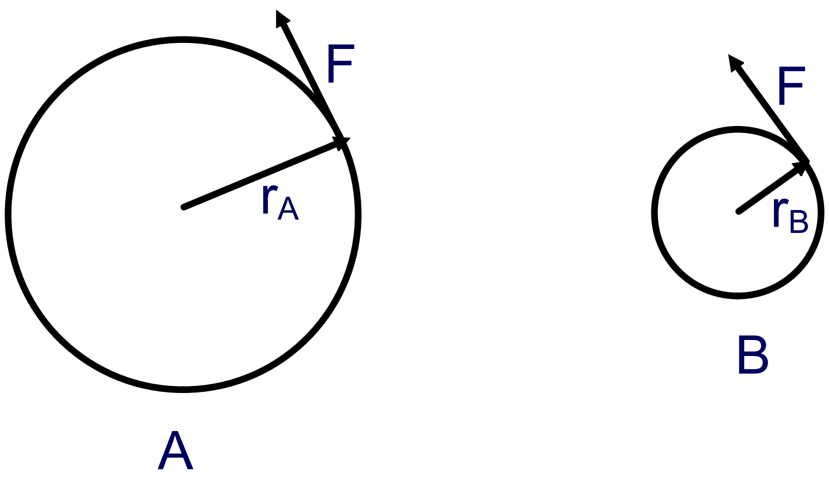
Assuming a constant force applied to both screwdrivers, which one would turn the screw easier? Would a longer shank help?



Screwdriver handle

The force is applied perpendicular to the radius of the handle. The forces are the same. Since $\tau = rF\sin\theta$, the greater the radius, the greater the torque, hence the greater angular acceleration.

Screwdriver A is more effective. The length of the shank does not matter (assuming it is perfectly rigid and does not bend or deform under the force).



39 Which statements regarding torque are true? Select two answers.

A Torque is most commonly measured in N-m

B Torque is most commonly measured in J

C Torque is a vector

D Torque is the same as work and energy

Answer

Torque

Torque is the rotational analog to Force.

$$\tau = rF\sin\theta$$

Newton's Second Law states that F = ma and has been applied to point particles that don't rotate. We now want to analyze extended rigid bodies that do rotate. Start with the force on one small piece of the rigid body as it's rotating:

$$F_i = m_i a_i$$

$$F_i = m_i r_i \alpha$$

since $a_i=r_i\alpha$, where r_i is the distance between the small piece of the object and the axis of rotation

Torque

Assume that the force is applied perpendicular to the line connected to the axis of rotation (sinθ=1)

$$\tau_i = r_i F_i$$

$$\tau_i = r_i (m_i r_i \alpha) \quad \text{from the previous page}$$

$$\tau_i = m_i r_i^2 \alpha$$

Now sum up all the little pieces of the rigid body:

$$\Sigma \tau_i = \left(\Sigma m_i r_i^2 \right) \alpha$$

Why was α factored out of the summation?

Because α is constant for all points on the rigid object.

Moment of Inertia

This looks very similar to Newton's Second Law in translational motion. ΣF has been replaced by $\Sigma\tau$ and a has been replaced by α. All good. But m has been replaced by Σmr^2.

This is a new concept; it is called the Moment of Inertia, I.

$$I = \Sigma m_i r_i^2$$

The mass of a given rigid object is always constant.

Can the same be said for the moment of inertia?

Moment of Inertia

No - it depends on the configuration of the rigid object, and where its axis of rotation is located.

Now, here's Newton's Second Law for rotational motion.

$$\Sigma\tau = I\alpha$$

where $I = \Sigma m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^3 + \cdots$

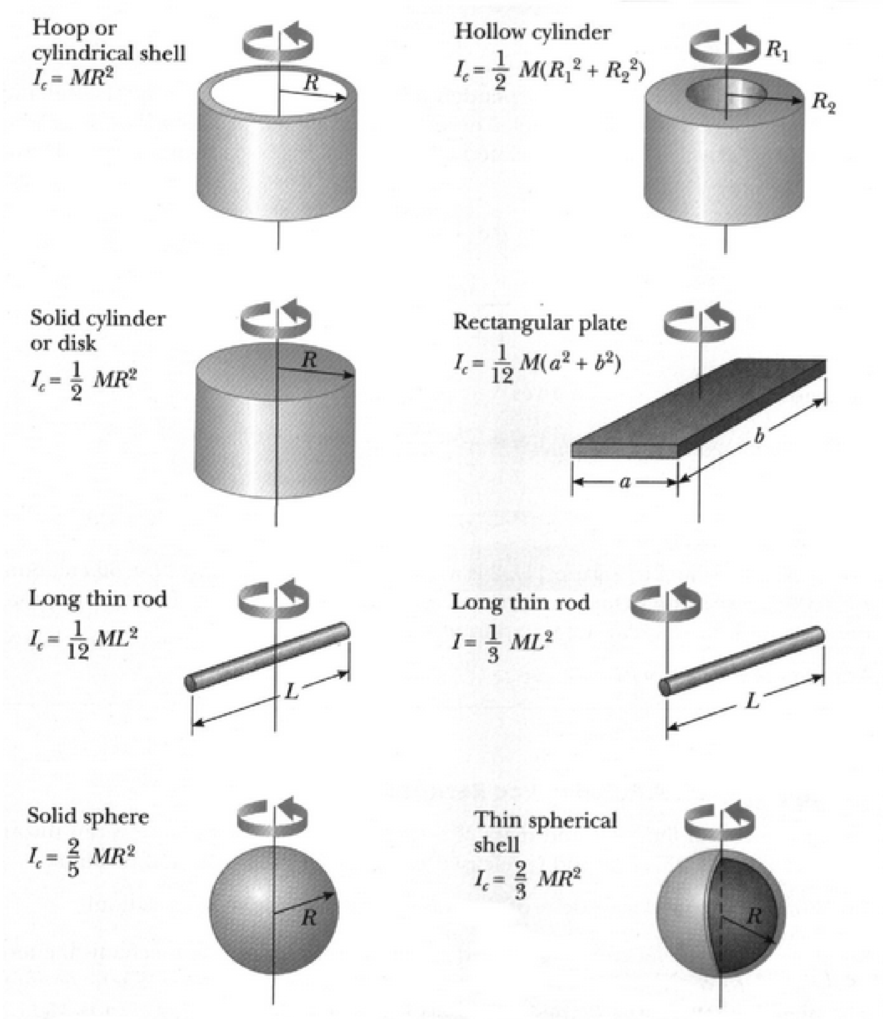
If more of the little pieces of the rotating object are located further away from the axis of rotation, then its moment of inertia will be greater than an object of the same mass where more of the mass is concentrated near the axis of rotation.

Moment of Inertia

Here are various moments of inertia for common objects.

All of the objects have the same mass, M, but different shapes and different axes of rotation.

Take some time to see how and why the moments of inertia change.



Moment of Inertia

Mass is defined as the resistance of an object to accelerate due to an applied force.

The moment of inertia is a measure of an object's resistance to angular acceleration due to an applied torque.

The greater the moment of inertia of an object, the less it will accelerate due to an applied torque.

The same torque is applied to a solid cylinder and a sphere, each with the same mass and radius.

Which object will have a greater angular acceleration? You can go back a slide to find their moments of inertia.

Moment of Inertia

Since the solid sphere has a smaller moment of inertia, it will have a greater angular acceleration.

The angular acceleration can be different for the same object, if a different axis of rotation is chosen.

If you want to rotate from rest, a long thin rod, where should you hold the rod to make it easier to rotate?

It should be held in the middle. By rotating it about that axis of rotation, the rod has a smaller moment of inertia, so for a given torque, it will have a greater angular acceleration - it will be easier to rotate.

40 A child rolls a 0.02 kg hoop with an angular acceleration of 10 rad/s^2. If the hoop has a radius of 1 m, what is the net torque on the hoop?

$$I_{hoop} = MR^2$$

A 0.2 N-m

B 0.3 N-m

C 0.4 N-m

D 0.5 N-m

Answer

41 A construction worker spins a square sheet of metal of mass 0.040 kg with an angular acceleration of 10.0 rad/s^2 on a vertical spindle (pin). What are the dimensions of the sheet if the net torque on the sheet is 1.00 N-m?

$$I_{rect} = \frac{1}{12}M\left(a^2 + b^2\right)$$

Answer

Parallel Axis Theorem

If the moment of inertia about the center of mass of a rigid body is known, the moment of inertia about any other axis of rotation of the body can be calculated by use of the Parallel Axis Theorem. Often, this simplifies the mathematics.

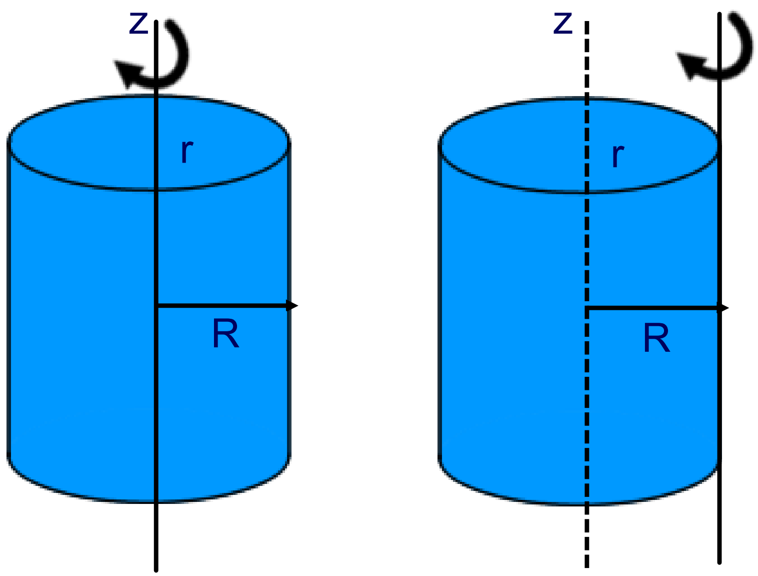
This theorem will be presented without proof:

$$I = I_{cm} + MD^2$$

where D is the distance between the new axis of rotation from the axis of rotation through the center of mass (I_{cm}), and M is the total mass of the object.

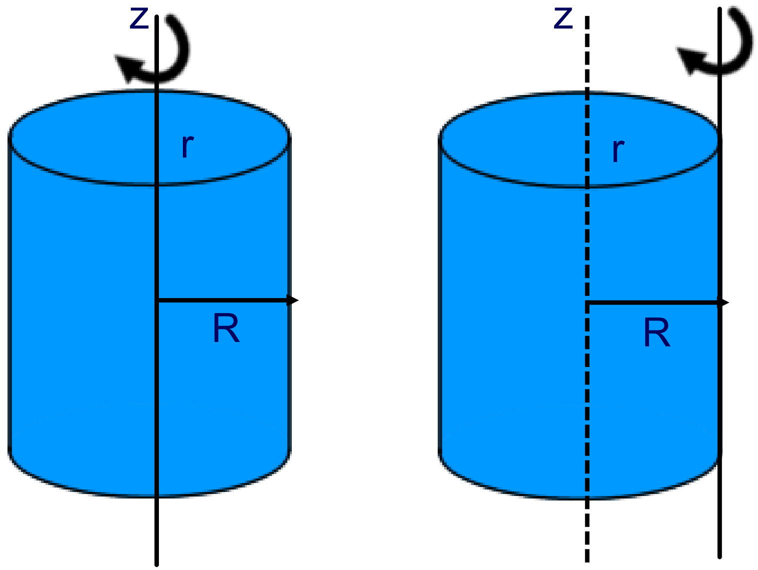
Parallel Axis Theorem

Apply this theorem to the solid cylinder of mass M and radius R.



The moment of inertia when the cylinder is rotated about its center of mass is $I = 1/2\ MR^2$. Find the moment of inertia when it is rotated on an axis parallel to its central axis and at a distance, R.

Parallel Axis Theorem

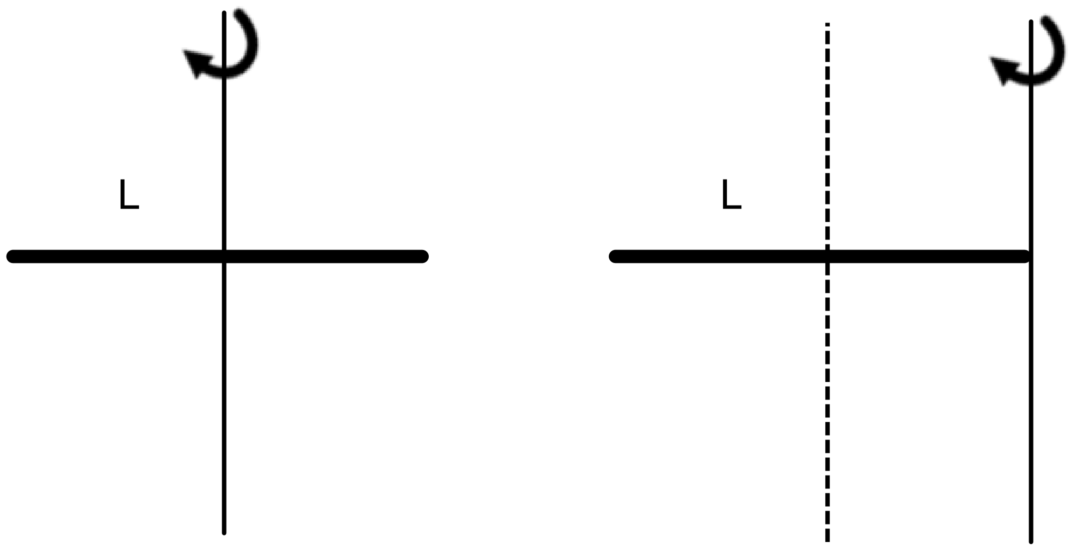


$$I = I_{cm} + MD^2$$

$$I = \frac{1}{2}MR^2 + MR^2$$

$$I = \frac{3}{2}MR^2$$

42 A long thin rod, of length, L, when rotated about an axis through its center of mass has a moment of inertia of $I = 1/12\ ML^2$. What is its moment of inertia when rotated about an axis through one of its endpoints?



Answer

Rotational Kinetic Energy

Return to Table of Contents

Rotational Kinetic Energy

After Kinematics and Dynamics were covered, your physics education was continued with the concept of Energy. Energy is hard to define, but since it is a scalar quantity, it is great for solving problems.

By now, you should be ready for the concept of a rotational version of the translational kinetic energy.

Given that $KE = 1/2\ mv^2$, what substitution should be made?

Rotational Kinetic Energy

Use the rotational equivalent of velocity and substitute it into the Kinetic Energy equation. The first column shows where one piece of a rotating object is being considered, and it is now a rotational energy since v is replaced by ω.

$$KE_T = \frac{1}{2}mv^2$$

$$KE_{R1} = \frac{1}{2}m_1\left(r_1\omega\right)^2$$

$$KE_{R1} = \frac{1}{2}m_1 r_1^{\ 2}\omega^2$$

Add up all the KE_{Ri}'s for each small mass

$$KE_R = \frac{1}{2}m_1 r_1^2\omega^2 + \frac{1}{2}m_2 r_2^2\omega^2 + \cdots$$

$$KE_R = \frac{1}{2}(\Sigma m_i r_i^{\ 2})\omega^2$$

$$KE_R = \frac{1}{2}I\omega^2$$

Work done by Torque

The concept of Work was introduced in parallel with Energy, where $W = F\Delta d_{parallel}$

A shortcut will now be taken to determine how much work is done by an applied torque - the rotational equivalents of force and displacement will just be substituted into the above equation:

$$W = \tau\Delta\theta$$

What fell out of the original Work equation and why?

The angular nature of the equation. Torque is, by definition, always perpendicular to the angular displacement, so $\sin\theta = 1$, and Work is the product of their amplitudes.

Total Kinetic Energy

This course has so far discussed three types of energy - GPE, KE and EPE (Elastic Potential Energy).

All three of those were dealt with in terms of a point mass - there was no rotation.

Now that we have rigid rotating bodies, KE will be redefined as a total kinetic energy that sums up the translational and rotational aspects.

$$KE_{total} = KE_{translational} + KE_{rotational}$$

$$KE_T = KE_t + KE_r$$

Total Kinetic Energy

A common problem in rotational motion is having two different objects, each with the same mass, and determining which will get to the bottom of an incline first.

If they were just blocks, or even spheres that slid down without rotating, they would get to the bottom at the same time, since objects fall at the same rate.

By using an extended mass (not a point particle), and rotation, it's quite different.

Why? Think about Energy. At the top of the ramp, both objects have the same energy - it's all potential. How about at the bottom?

Total Kinetic Energy

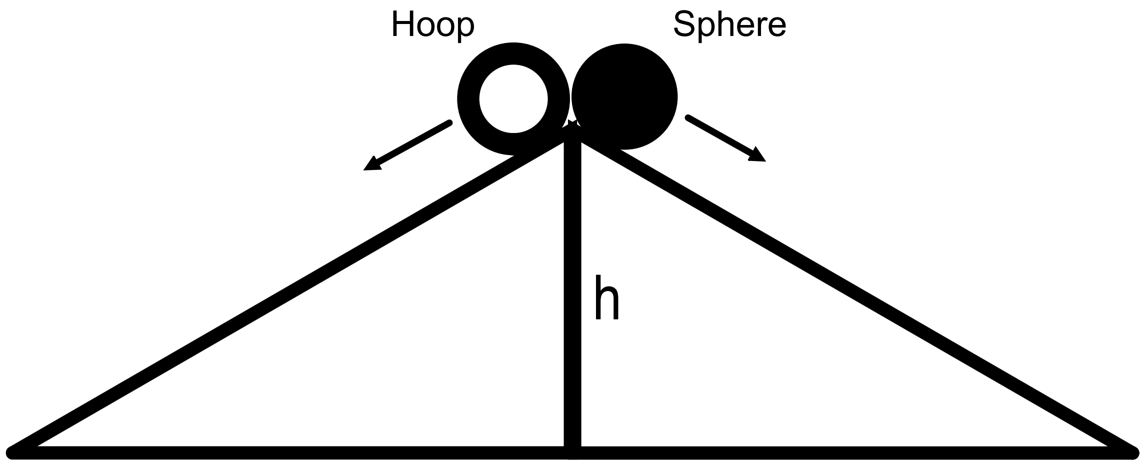
For a non rotating object, all of the gravitational potential energy is transferred into translational kinetic energy. If the object rotates, then part of that energy goes into rotational kinetic energy.

Rotational kinetic energy depends on the shape of the object - which affects the moment of inertia.

A sphere and a hoop of the same mass and radius both start at rest, at the top of a ramp. Assume that both objects roll without slipping, which one gets to the bottom first?

Total Kinetic Energy

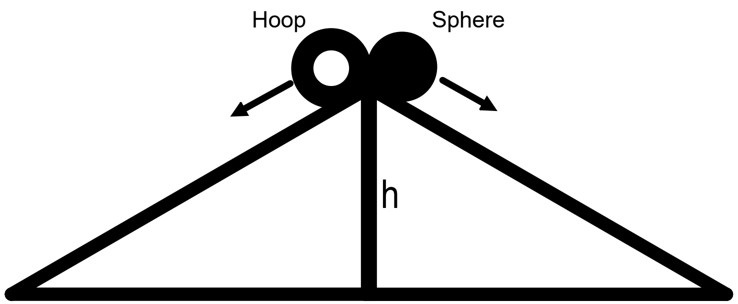
Here's the sketch.



What is the equation, using the Conservation of Energy, that will give the velocity of the objects at the bottom of the ramp? Solve the equation. Qualitatively, how can that be used to determine which object reaches the bottom first?

Try working this in your groups before going to the next slide.

Total Kinetic Energy



$$\left(KE_T + GPE\right)_0 = \left(KE_T + GPE\right)_f$$

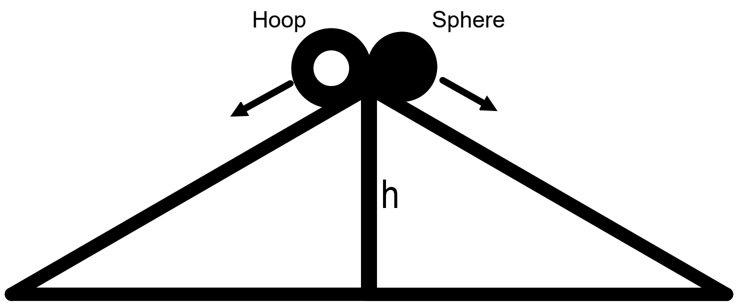
$$\left(KE_t + KE_r + GPE\right)_0 = \left(KE_t + KE_r + GPE\right)_f$$

$$GPE_0 = \left(KE_t + KE_r\right)_f$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

This equation works for both the hoop and the sphere.

Total Kinetic Energy



$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

Did you question why it was stated that the sphere and the hoop roll without slipping? When that occurs, then $v = r\omega$. We will use this fact to replace ω in the above equation with v/r.

If an object is sliding (slipping) without rotating, then this isn't true. Why? Think of a car on an icy road - when the driver hits the brakes quickly, what happens?

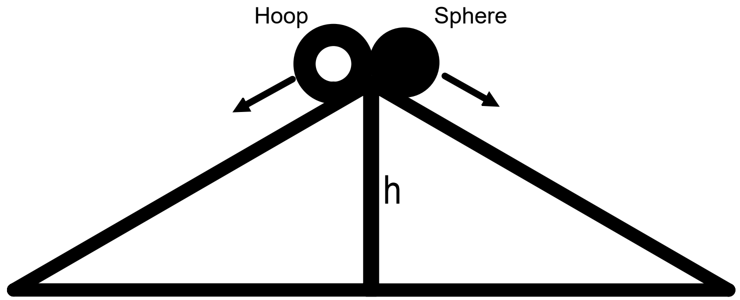
Total Kinetic Energy

The wheels lock up (although anti-lock brakes help to minimize that), and the car wheels slip on the road. The wheels don't rotate anymore - the car is just sliding.

So, the wheels have a translational velocity, v, but the angular velocity of the wheels, $\omega = 0$. Clearly $v \neq r\omega$.

Back to the problem, where the substitution $\omega = v/r$ will be made in the rotational kinetic energy expression.

Total Kinetic Energy



Hoop

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}(MR^2)\left(\frac{v}{R}\right)^2$$

$$2gh = v^2 + v^2$$

$$v = \sqrt{gh}$$

Sphere

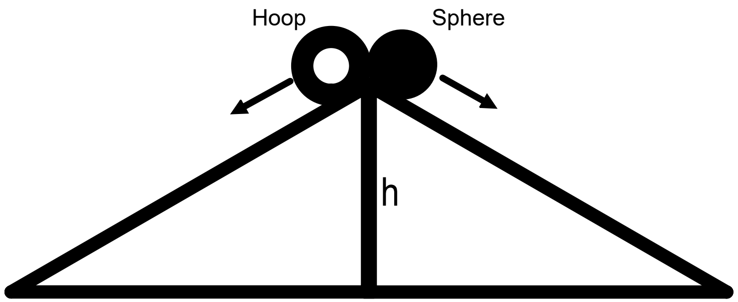
$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

$$2gh = v^2 + \frac{2}{5}v^2$$

$$v = \sqrt{\left(\frac{10}{7}\right)gh} = 1.2\sqrt{gh}$$

Total Kinetic Energy

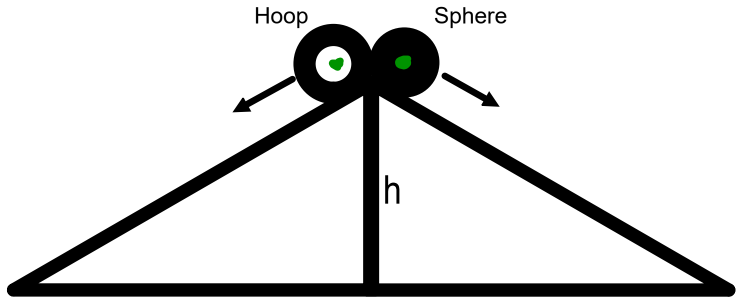


The velocity of the sphere is 1.2 times as much as the velocity of the hoop. Did that surprise you?

Qualitatively, that means that the sphere gets to the bottom of the ramp first - because at any point on the incline, the sphere has a greater velocity (h is just the height above ground). Solving the kinematics equations would give the same answer.

One more question - why did this happen?

Total Kinetic Energy



Both the sphere and the hoop started with the same GPE. And, at the bottom of the ramp, both had zero GPE and a maximum KE - again, both the same.

But - the hoop, with its greater moment of inertia, took a greater amount of the total kinetic energy to make it rotate - so there was less kinetic energy available for translational motion.

Hence, the sphere has a greater translational velocity than the hoop and reaches the bottom first.

43 A spherical ball with a radius of 0.50 m rolls, without slipping, down a 10.0 m high hill. What is the velocity of the ball at the bottom of the hill? Use $g = 10\ m/s^2$.

$$I = \frac{2}{5}MR^2$$

A 10 m/s

B 11 m/s

C 12 m/s

D 13 m/s

Answer

44 A solid cylinder with a radius of 0.50 m rolls, without slipping, down a hill. What is the height of the hill if the final velocity of the block is 10.0 m/s? Use g = 10 m/s^2.

$$I = \frac{1}{2}MR^2$$

Answer

45 Which change would lower the final velocity of a spherical ball rolling down a hill? Select two answers.

A Changing the spherical ball to a solid cylinder

B Increasing the mass of the ball

C Decreasing the mass of the ball

D Decreasing the height of the hill

Answer

Angular Momentum

Return to Table
of Contents

Angular Momentum

As was done in the last chapter, we won't derive angular momentum from linear momentum - we're just going to substitute in the angular values for the linear (translational) ones.

Linear momentum (p): $p = mv$

Angular momentum (L): $L = I\omega$

Conservation of Angular Momentum

The conservation of (linear) momentum states that in the absence of external forces, momentum is conserved. This came from the original statement of Newton's Second Law:

$$\Sigma F = \frac{\Delta p}{\Delta t}$$

If there are no external forces, then:

$$\Sigma F = \frac{\Delta p}{\Delta t} = 0$$

$$\Delta p = 0$$

There is no change in momentum - it is conserved.

Conservation of Angular Momentum

Newton's Second Law (rotational version):

$$\Sigma \tau = \frac{\Delta L}{\Delta t}$$

If there are no external torques, then:

$$\Sigma \tau = \frac{\Delta L}{\Delta t} = 0$$

$$\Delta L = 0$$

There is no change in angular momentum - it is conserved.

Conservation of Angular Momentum

The standard example used to illustrate the conservation of angular momentum is the ice skater. Start by assuming a frictionless surface - ice is pretty close to that - although if it were totally frictionless, the skater would not be able to stand up and skate!

The skater starts with his arms stretched out and spins around in place. He then pulls in his arms.

What happens to his rotational velocity after he pulls in his arms?

After you discuss this in your groups, please click on the below and see if you were correct. Why did this happen?

Click here to see the ice skater in action

Conservation of Angular Momentum

By assuming a nearly frictionless surface, that implies there is no net external torque on the skater, so the conservation of angular momentum can be used.

$$\Sigma\tau = \frac{\Delta L}{\Delta t} = 0$$

$$\Delta L = 0$$

$$L_0 = L_f$$

$$I_0\omega_0 = I_f\omega_f$$

$$\omega_f = \frac{I_0}{I_f}\omega_0$$

The human body is not a rigid sphere or a hoop or anything simple. But, knowing what the definition of moment of inertia is, at what point in the spinning does the skater have a greater moment of inertia?

Conservation of Angular Momentum

When the skater's arms are stretched out, more of his mass is located further from his axis of rotation (the skates on the ice), so he has a greater moment of inertia then when he tucks his arms in. In the video, I_0 is greater than I_f. Thus, ω_f is greater than ω_0 and the skater rotates faster with his arms tucked in.

$$\omega_f = \frac{I_0}{I_f}\omega_0$$

46 A 2.0 kg solid spherical ball with a radius of 0.20 m rolls with an angular velocity of 5.0 rad/s down a street. Magneto shrinks the radius of the ball down to 0.10 m, but does not change the ball's mass and keeps it rolling. What is the new angular velocity of the ball?

Answer

47 An ice skater extends her 0.5 m arms out and begins to spin. Then she brings her arms back to her chest. Which statements are true in this scenario? Select two answers.

A She increases her angular velocity.

B She decreases her angular velocity.

C Her moment of inertia decreases as she brings her arms in.

D Her moment of inertia increases as she brings her arms in.

Answer