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# **AP Physics 1**

## **Momentum**

**2017-07-20**

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# Momentum

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# Momentum Defined

Newton's First Law tells us that objects remain in motion with a constant velocity unless acted upon by an external force (Law of Inertia).

In our experience:

- When objects of different mass travel with the same velocity, the one with more mass is harder to stop.
- When objects of equal mass travel with different velocities, the faster one is harder to stop.

# Momentum Defined

Define a new quantity, *momentum* ( $p$ ), that takes these observations into account:

$$\textit{momentum} = \textit{mass} \times \textit{velocity}$$

$$p = mv$$

*What's wrong with this equation?*

# Momentum is a Vector Quantity

Since:

- mass is a scalar quantity
- velocity is a vector quantity
- the product of a scalar and a vector is a vector

and:

- *momentum = mass × velocity*

therefore:

Momentum is a vector quantity - it has magnitude and direction

$$\vec{p} = m\vec{v}$$

# SI Unit for Momentum

There is no specially named unit for momentum - so there is an opportunity for it to be named after a renowned physicist!

We use the product of the units of mass and velocity.

mass x velocity



**kg·m/s**

Since momentum is a vector, you need to specify a direction - such as to the right, up, down, to the east, or use positive (to the right) or negative (to the left), or any direction that is relevant to the problem.

1 Which object or objects have the greatest momentum?

- A A large trailer truck moving at 30 m/s.
- B A motorcycle moving at 30 m/s.
- C The Empire State Building.
- D Choices A and B have the same momentum.
- I need help

2 What is the momentum of a 20 kg object moving to the right with a velocity of 5 m/s?

- 4 kgm/s
- 5 kgm/s
- 20 kgm/s
- 100 kgm/s
- I need help

3 What is the momentum of a 20 kg object moving to the left with a velocity of 5 m/s?

- 4 kgm/s
- 5 kgm/s
- 20 kgm/s
- 100 kgm/s
- I need help

4 What is the velocity of a 5 kg object whose momentum is  $-15 \text{ kg}\cdot\text{m/s}$ ?

A  $-3 \text{ m/s}$

B  $-5 \text{ m/s}$

C  $-15 \text{ m/s}$

D  $-60 \text{ m/s}$

E I need help



5 What is the mass of an object that has a momentum of 35 kg-m/s with a velocity of 7 m/s?

5 kg

7 kg

15 kg

21 kg

I need help

# Impulse-Momentum Equation

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# Change in Momentum

Suppose that there is an event that changes an object's momentum.

- from  $\vec{p}_0$  - the initial momentum (just before the event)
- by  $\Delta\vec{p}$  - the change in momentum
- to  $\vec{p}_f$  - the final momentum (just after the event)

The equation for momentum change is:

$$\vec{p}_0 + \Delta\vec{p} = \vec{p}_f$$

# Momentum Change = Impulse

*Momentum change equation:*

$$\vec{p}_0 + \Delta\vec{p} = \vec{p}_f$$

*Newton's First Law* tells us that the velocity (and so the momentum) of an object won't change unless the object is affected by an external force.

*Look at the above equation. Can you relate Newton's First Law to the  $\Delta\vec{p}$  term?*

$\Delta\vec{p}$  is related to the external force.

# Momentum Change = Impulse

Let's now use Newton's Second Law , and see if we can relate  $\Delta\vec{p}$  to the external force in an explicit manner:

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m \frac{\Delta\vec{v}}{\Delta t} = m \frac{(\vec{v}_f - \vec{v}_0)}{\Delta t}$$

$$\vec{F}\Delta t = m\vec{v}_f - m\vec{v}_0 = \vec{p}_f - \vec{p}_0$$

$$\vec{F}\Delta t = \Delta\vec{p}$$

We've found that the momentum change of an object is equal to an external Force applied to the object over a period of time.

# Impulse Momentum Equation

The force acting over a period of time on an object is now defined as Impulse and we have the Impulse-Momentum equation:

$$\vec{I} = \vec{F} \Delta t = \Delta \vec{p}$$

# SI Unit for Impulse

There no special unit for impulse.

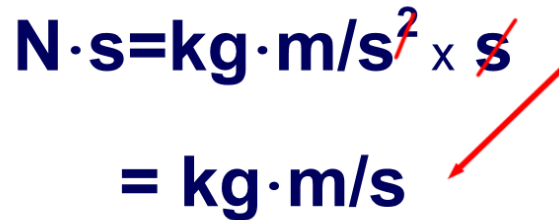
We use the product of the units of force and time.

force x time



**N·s**

Recall that  $N = \text{kg} \cdot \text{m}/\text{s}^2$ , so

$$\begin{aligned} \mathbf{N \cdot s} &= \mathbf{kg \cdot m/s^2} \times \mathbf{s} \\ &= \mathbf{kg \cdot m/s} \end{aligned}$$


This is also the unit for momentum, which is a good thing since Impulse is the change in momentum.

6 There is a battery powered wheeled cart moving towards you at a constant velocity. You want to apply a force to the cart to move it in the opposite direction. Which combination of the following variables will result in the greatest change of momentum for the cart? **Select two answers.**

- Increase the applied force.
- Increase the time that the force is applied.
- Maintain the same applied force.
- Decrease the time that the force is applied.
- E I need help



7 From which law or principle is the Impulse-Momentum equation derived from?

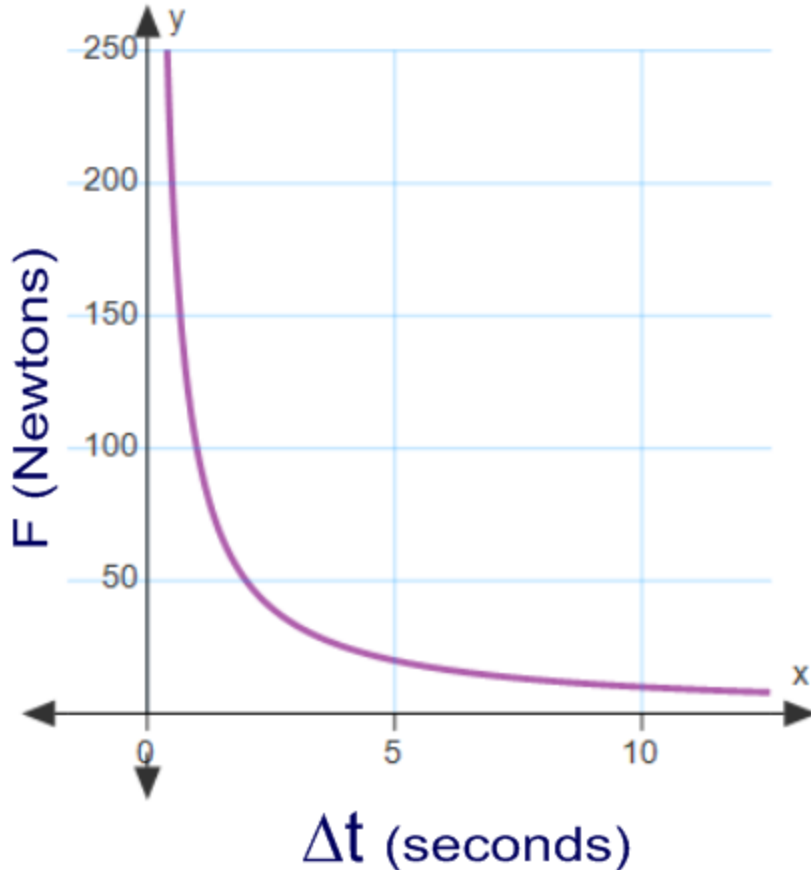
- A Conservation of Energy.
- B Newton's First Law.
- C Newton's Second Law.
- D Conservation of Momentum.
- I need help

8 Can the impulse applied to an object be negative? Why or why not? Give an example to explain your answer.

**Answer**

# Effect of Collision Time on Force

$$\text{Impulse} = \mathbf{F}\Delta t = F\Delta t$$



Since force is inversely proportional to  $\Delta t$ , changing the  $\Delta t$  of a given impulse by a small amount can greatly change the force exerted on an object!

Let's just work with the magnitudes of the impulse and momentum, so the vector sign will be dropped.

# Every Day Applications

$$\text{Impulse} = \mathbf{F}\Delta t = F\Delta t$$

The inverse relationship of force and time interval leads to many interesting applications of the Impulse-Momentum equation to everyday experiences such as:

- car structural safety design
- car air bags
- landing after parachuting
- martial arts
- hitting a baseball
- catching a baseball

# Every Day Applications

$$\mathbf{F}\Delta t = \mathbf{F}\Delta t$$

Let's analyze two specific cases from the previous list:

- car air bags
- hitting a baseball

Whenever you have an equation with three variables, you have to decide which value will be fixed, and which will be varied, to determine the impact on the third value.

For the car air bags, we'll fix  $\Delta\mathbf{p}$ , vary  $\Delta t$  and see its impact on  $\mathbf{F}$ .

For the bat hitting a ball, we'll fix  $\mathbf{F}$ , vary  $\Delta t$  and see the impact on  $\Delta\mathbf{p}$ .

# Car Air Bags

$$F\Delta t = F\Delta t$$

In the Dynamics unit of this course, it was shown how during an accident, seat belts protect passengers from the effect of Newton's First Law by stopping the passenger with the car, and preventing them from striking the dashboard and window.

They also provide another benefit explained by the Impulse-Momentum equation. But, this benefit is greatly enhanced by the presence of air bags. Can you see what this benefit is?

# Car Air Bags

$$F\Delta t = F\Delta t$$

The seat belt also increases the time interval that it takes the passenger to slow down - there is some play in the seat belt, that allows you to move forward a bit before it stops you.

The Air bag will increase that time interval much more than the seat belt by rapidly expanding, letting the passenger strike it, then deflating.

Earlier it was stated that for the Air bag example,  $\Delta p$  would be fixed and  $\Delta t$  would be varied. So, we've just increased  $\Delta t$ . Why is  $\Delta p$  fixed?

# Car Air Bags

$$\mathbf{F}\Delta t = \mathbf{F}\Delta t$$

$\Delta p$  is fixed, because as long as the passenger remains in the car, the car (and the passengers) started with a certain velocity and finished with a final velocity of zero, independent of seat belts or air bags.

Rearranging the equation, we have:

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

F represents the Force delivered to the passenger due to the accident.



# Car Air Bags

$$F\Delta t = F\Delta t$$

Since  $\Delta p$  is fixed, by extending the time interval ( $\Delta t$  increases) that it takes a passenger to come to rest (seat belt and air bag), the force,  $F$  delivered to the passenger is smaller.

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

Less force on the passenger means less physical harm. Also, another benefit needs a quick discussion of Pressure.

Pressure is Force per unit area. By increasing the area of the body that feels the force (the air bag is big), less pressure is delivered to parts of the body - reducing the chance of puncturing the body. Also good.

9 If a car is in a front end collision, which of the below factors will help reduce the injury to the driver and passengers? **Select two answers.**

- An absolutely rigid car body that doesn't deform.
- Deployment of an air bag for each adult in the car.
- Deployment of an air bag only for the driver.
- The proper wearing of a seatbelt or child seat for each person in the car.
- E I need help

10 Which of the following variables are fixed, using the Impulse - Momentum equation, when analyzing a moving car that strikes a barrier and comes to rest?

- A Force delivered to the passengers.
- B Interval of time before the passengers come to rest.
- C Momentum change of the car.
- D Acceleration of the passengers within the car.
- I need help

# Hitting a Baseball

$$\mathbf{F}\Delta t = \Delta\mathbf{p}$$

Now, we're going to take a case where there is a given force, and the time interval will be varied to find out what happens to the change of momentum.

This is different from the Air Bag example just worked ( $\Delta\mathbf{p}$  was constant,  $\Delta t$  was varied, and its impact on  $\mathbf{F}$  was found).

Assume a baseball batter swings with a given force that is applied over the interval of the ball in contact with the bat. The ball is approaching the batter with a positive momentum.

What is the goal of the batter?

# Hitting a Baseball

$$\mathbf{F}\Delta t = \mathbf{F}\Delta t$$

The batter wants to hit the ball and get the largest  $\Delta\mathbf{p}$  possible for his force which depends on his strength and timing. The greater the  $\Delta\mathbf{p}$ , the faster the ball will fly off his bat, which will result in it going further, hopefully to the seats.

Hitting a baseball is way more complex than the analysis that will follow. If you're interested in more information, please check out the book, *The Physics of Baseball*, written by Robert Adair, a Yale University physicist.

# Hitting a Baseball

$$\mathbf{F}\Delta t = \mathbf{F}\Delta t$$

In this case, the batter wants to maximize the time that his bat (which is providing the force) is in contact with the ball. This means he should follow through with his swing.

$$\Delta\vec{p} = \vec{F}\Delta t$$

The batter needs to apply a large impulse to reverse the ball's large momentum from the positive direction (towards the batter) to the negative direction (heading towards the center field bleachers).

# Every Day Applications

$$F\Delta t = F\Delta t$$

Now, discuss the other examples. Make sure you decide which object in the collision is more "affected" by the force or the change in momentum, and which variables are capable of being varied.

Consider the Air Bag example - the car experiences the same impulse as the passenger during an accident, but a car is less valuable than a human being - so it is more important for the passenger that less force is delivered to his body - and more force is absorbed by the car.

- car structural safety design
- landing after parachuting
- martial arts
- catching a baseball

11 An external force of 25 N acts on a system for 10.0 s. What is the magnitude of the impulse delivered to the system?

10 Ns

25 Ns

50 Ns

250 Ns

I need help



12 An external force of 25 N acts on a system for 10.0 s. The impulse delivered was 250 N-s. What is the magnitude of the change in momentum of the system?

- A 10 Ns
- B 25 Ns
- C 50 Ns
- D 250 Ns
- E I need help

13 The momentum change of an object is equal to the \_\_\_\_\_.

- A force acting on the object.
- B impulse acting on the object.
- C velocity change of the object.
- D object's mass times the force acting it.
- E I need help

14 Air bags are used in cars because they:

- A increase the force with which a passenger hits the dashboard.
- B increase the duration (time) of the passenger's impact.
- C decrease the change in momentum of a collision.
- D decrease the impulse in a collision.
- E I need help

15 One car crashes into a concrete barrier. Another car crashes into a collapsible barrier at the same speed. What is the difference between the two crashes? **Select two answers.**

- A change in momentum
- B force on the passengers
- C impact time on the passengers
- D final momentum
- E I need help

16 In order to increase the final momentum of a golf ball, the golfer should: **Select two answers:**

- A maintain the speed of the golf club after the impact (follow through).
- B Hit the ball with a greater force.
- C Decrease the time of contact between the club and the ball.
- D Decrease the initial momentum of the golf club.
- I need help

17 An external force acts on an object for 0.0020 s. During that time the object's momentum increases by 400 kg-m/s. What was the magnitude of the force?

8,000 N

25,000 N

40,000 N

200,000 N

I need help

18 A 50,000 N force acts for 0.030 s on a 2.5 kg object that was initially at rest. What is its final velocity?

- 600 m/s
- 1200 m/s
- 1400 m/s
- 1800 m/s
- I need help

19 A 1200 kg car slows from 40.0 m/s to 5.0 m/s in 2.0 s.  
What force was applied by the brakes to slow the car?

- A -13,000 N
- B -21,000 N
- C -48,000 N
- D -55,000 N
- E I need help



# Graphical Interpretation of Impulse

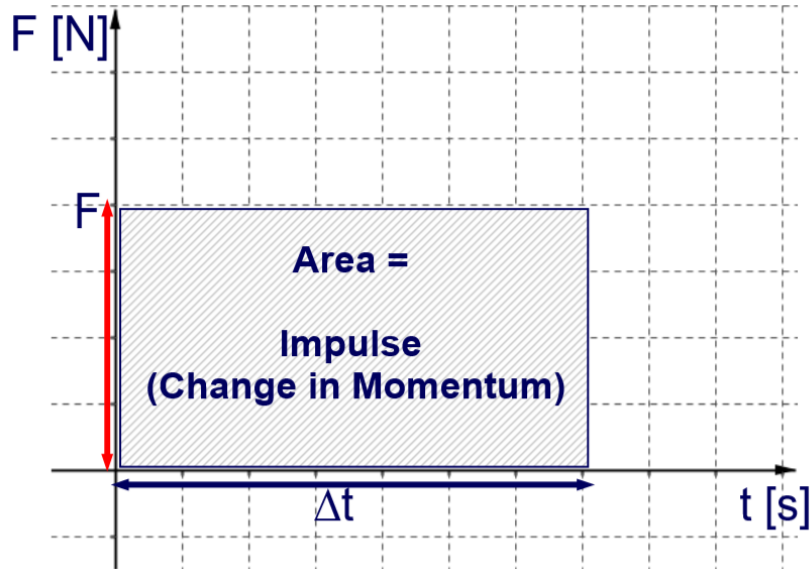
Graphs are powerful tools to help solve physics problems.

So far, we've dealt with a constant force exerted over a given time interval. But forces are not always constant - most of the time, they are changing over time.

In the baseball example, the force of the bat starts out very small as it makes initial contact with the ball. It then rapidly rises to its maximum value, then decreases as the ball leaves the bat. This would be a very difficult problem to handle without a graph.

Start with representing a constant force over a time interval,  $\Delta t$ , and plot Force vs. time on a graph.

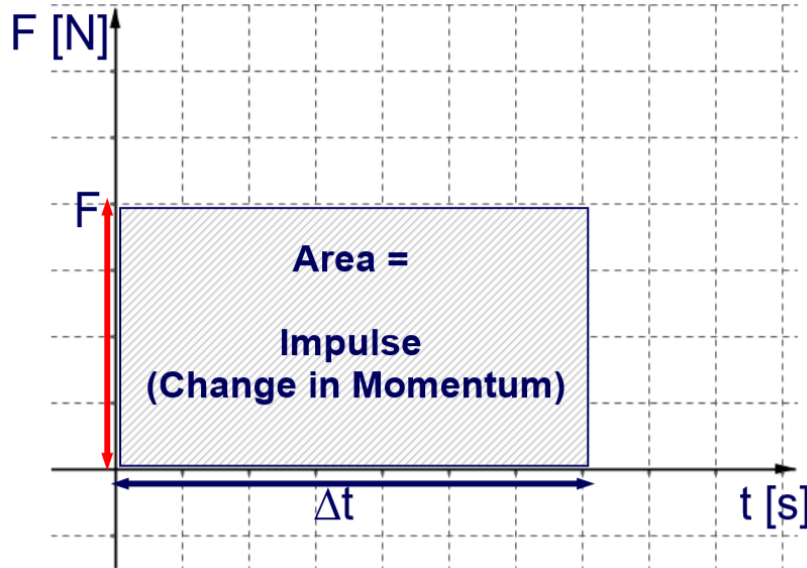
# Graphical Interpretation of Impulse



Note, the area under the Force-Time graph is:

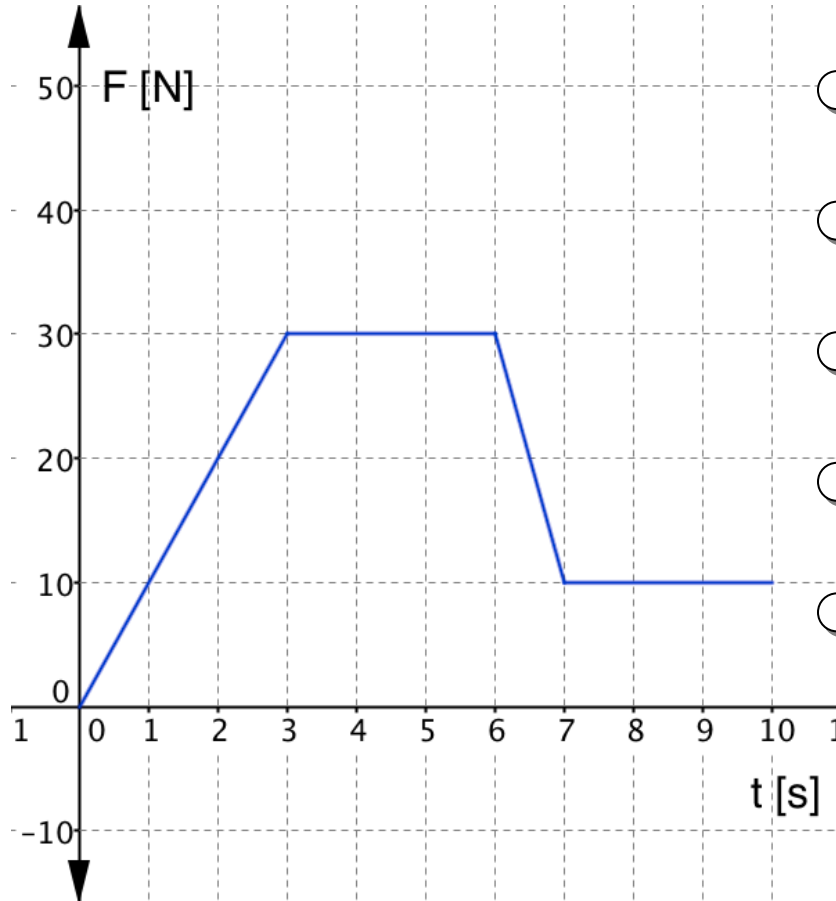
$$\begin{aligned} \text{height} \times \text{base} &= F \Delta t \\ &= I \quad (\text{impulse}) \\ &= \Delta p \quad (\text{change in momentum}) \end{aligned}$$

# Graphical Interpretation of Impulse



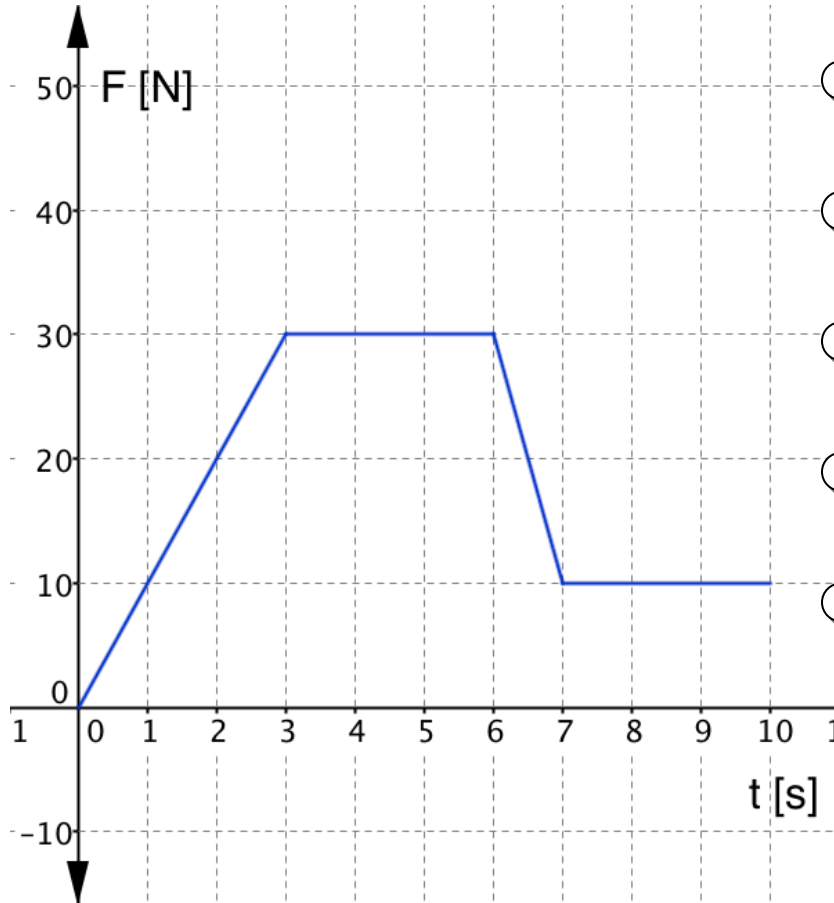
By taking the area under a Force-time graph, we have found both Impulse and  $\Delta p$ . If the force is not constant, the area can be found by breaking it into solvable shapes (rectangles, triangles) and summing them up.

20 Using the F-t graph shown, what is the change in momentum during the time interval from 0 to 6.0 s?



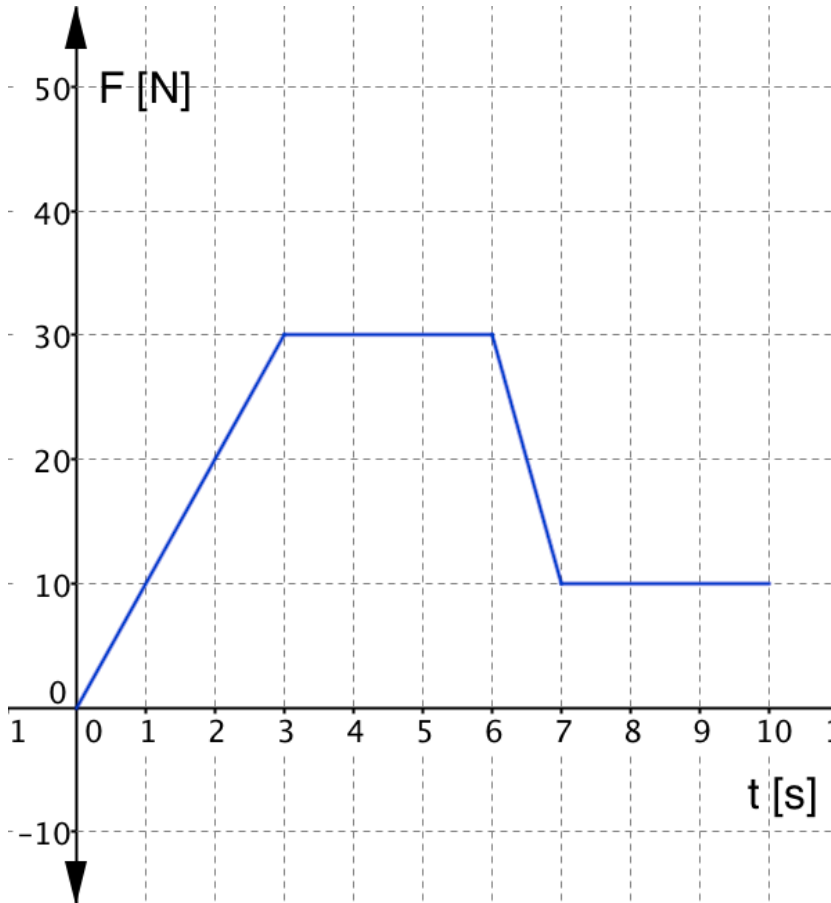
- A 100 Ns
- B 135 Ns
- C 150 Ns
- D 165 Ns
- E I need help

21 A 5.0 kg object with an initial velocity of 3.0 m/s experiences the force shown in the graph. What is its velocity at 6.0 s?



- A 15 m/s
- B 18 m/s
- C 30 m/s
- D 32 m/s
- E I need help

22 Using the F-t graph shown, what is the change in momentum during the time interval from 6.0 to 10 s?



- 30 Ns
- 50 Ns
- 60 Ns
- 75 Ns
- I need help

# **The Momentum of a System of Objects**

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# The Momentum of a System of Objects

If a system contains more than one object, its total momentum is the *vector sum* of the momenta of those objects.

$$\vec{p}_{system} = \Sigma \vec{p}_i$$

$$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \dots$$

$$\vec{p}_{system} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \dots$$

*It's critically important to note that momenta add as vectors, not as scalars.*



# The Momentum of a System of Objects

$$\vec{p}_{system} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \dots$$

In order to determine the total momentum of a system:

- Determine a direction to be considered positive.
- Assign positive values to momenta in that direction.
- Assign negative values to momenta in the opposite direction.



Add the momenta to get the total momentum of the system.

# Example

Determine the momentum of a system of two objects:  $m_1$ , has a mass of 6 kg and a velocity of 13 m/s towards the east and  $m_2$ , has a mass of 14 kg and a velocity of 7 m/s towards the west.



$$m_1 = 6 \text{ kg}$$
$$v_1 = 13 \text{ m/s}$$



$$m_2 = 14 \text{ kg}$$
$$v_2 = -7 \text{ m/s}$$



$$\vec{p}_{system} = \Sigma \vec{p}_i$$

$$\vec{p}_{system} = \vec{p}_1 + \vec{p}_2$$

$$\vec{p}_{system} = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

$$\vec{p}_{system} = m_1 \vec{v}_1 + m_2 \vec{v}_2$$

$$\vec{p}_{system} = (6\text{kg})(13\text{ m/s}) + (14\text{kg})(-7\text{ m/s})$$

$$\vec{p}_{system} = -20\text{kg} \cdot \text{m/s}$$

23 Determine the momentum of a system of two objects:  
 $m_1$ , has a mass of 6.0 kg and a velocity of 20 m/s north,  
and  $m_2$ , has a mass of 3 kg and a velocity 20 m/s south.

- 60 kg m/s
- 30 kg m/s
- +30 kg m/s
- +60 kg m/s
- I need help

24 Determine the momentum of a system of two objects: the first has a mass of 8.0 kg and a velocity of 8.0 m/s to the east while the second has a mass of 5.0 kg and a velocity of 15 m/s to the west.

- A -22 kg m/s
- B -11 kg m/s
- C +11 kg m/s
- D +22 kg m/s
- E I need help

25 Determine the momentum of a system of three objects: The first has a mass of 7.0 kg and a velocity of 23 m/s north; the second has a mass of 9.0 kg and a velocity of 7.0 m/s north; and the third has a mass of 5.0 kg and a velocity of 42 m/s south.

- 14 kg m/s
- 7.0 kg m/s
- +7.0 kg m/s
- +14 kg m/s
- I need help

# Conservation of Momentum

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# Conservation Laws

Some of the most powerful concepts in science are called "conservation laws". This was covered in the Work and Energy unit - please refer back to that for more detail. Here is a summary.

Conservation laws:

- apply to closed systems - where the objects only interact with each other and nothing else.
- enable us to solve problems without worrying about the details of an event.

# Momentum is Conserved

In the last unit we learned that energy is conserved.

Like energy, momentum is a conserved property of nature. This means:

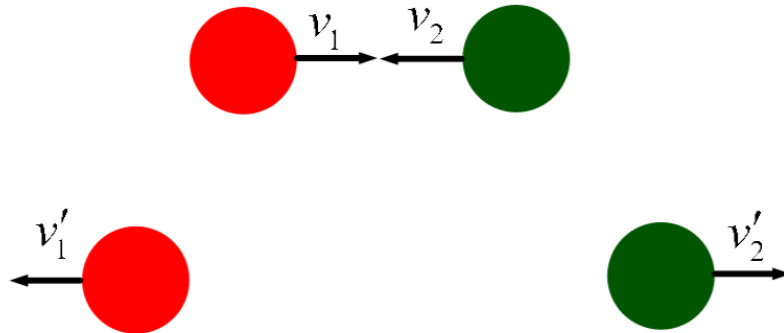
- Momentum is not created or destroyed.
- The total momentum in a closed system is always the same.
- *The only way the momentum of a system can change is if momentum is added or taken away by an outside force.*



# Conservation of Momentum

Conservation of Momentum is used to explain and predict the motion of a system of objects.

As with energy, it will only be necessary to compare the system at two times: just before and just after an event.



Here are two spheres colliding with each other and rebounding. If there are no external forces (this is a closed system), then the momentum before and after the collision is the same.

# Conservation of Momentum

The momentum of an object changes when it experiences an impulse ( $I = F\Delta t$ ):

$$\vec{p}_0 + \vec{I} = \vec{p}_f$$

This impulse arises when a non-zero external force acts on the object.

This is exactly the same for a system of objects.

$$\vec{p}_{system,0} + \vec{I} = \vec{p}_{system,f}$$

# Conservation of Momentum

If there is no net external force on the system, then  $\mathbf{F} = 0$ , and since  $\mathbf{I} = \mathbf{F}\Delta t$ ,  $\mathbf{I}$  is also zero:

$$\vec{p}_{system,0} + \vec{I} = \vec{p}_{system,f}$$

$$\vec{p}_{system,0} = \vec{p}_{system,f}$$

Since  $\mathbf{p}_{system,0} = \mathbf{p}_{system,f}$ , the momentum of the system is conserved, that is, it does not change.

26 Why don't the internal forces of a system change the momentum of the system?

**Answer**

27 An external, positive force acts on a system of objects. Which of the following are true? **Select two answers.**

A The velocity of the system remains the same.

B The velocity of the system increases.

C The momentum of the system decreases.

D The momentum of the system increases.

E I need help

# Types of Collisions

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# Types of Collisions

Objects in an isolated system can interact with each other in two basic ways:

- They can collide.
- If they are stuck together, they can explode (push apart).

In an isolated system both momentum and total energy are conserved. But the energy can change from one form to another.

Conservation of momentum and change in kinetic energy predict what will happen in these events.

# Types of Collisions

We differentiate collisions and explosions by the way the energy changes or does not change form.

- inelastic collisions: two objects collide, converting some kinetic energy into other forms of energy such as potential energy, heat or sound.
- elastic collisions: two objects collide and bounce off each other while conserving kinetic energy - energy is not transformed into any other type.
- explosions: an object or objects breaks apart because potential energy stored in one or more of the objects is transformed into kinetic energy.



# Inelastic Collisions

There are two types of Inelastic Collisions.

- perfect inelastic collisions: two objects collide, stick together and move as one mass after the collision, transferring some of the kinetic energy into other forms of energy.
- general inelastic collisions: two objects collide and bounce off each other, transferring some of the kinetic energy into other forms of energy.

# Elastic Collisions

There is really no such thing as a perfect elastic collision. During all collisions, some kinetic energy is always transformed into other forms of energy.

But some collisions transform so little energy away from kinetic energy that they can be dealt with as perfect elastic collisions.

In chemistry, the collisions between molecules and atoms are modeled as perfect elastic collisions to derive the Ideal Gas Law.

Other examples include a steel ball bearing dropping on a steel plate, a rubber "superball" bouncing on the ground, and billiard balls bouncing off each other.

# Explosions

A firecracker is an example of an explosion. The chemical potential energy inside the firecracker is transformed into kinetic energy, light and sound.

A cart with a compressed spring is a good example. When the spring is against a wall, and it is released, the cart starts moving - converting elastic potential energy into kinetic energy and sound.

*Think for a moment - can you see a resemblance between this phenomenon and either an elastic or inelastic collision?*

# Explosions

In both an inelastic collision and an explosion, energy changes between kinetic and potential energy. But they are time reversed!

An inelastic collision transforms kinetic energy into other forms of energy, such as potential energy.

An explosion changes potential energy into kinetic energy.

*Thus, the equations to predict their motion will be inverted.*

# Collisions and Explosions

<b>Event</b>	<b>Description</b>	<b>Momentum Conserved?</b>	<b>Kinetic Energy Conserved?</b>
General Inelastic Collision	Objects bounce off each other	Yes	No. Kinetic energy is converted to other forms of energy
Perfect Inelastic Collision	Objects stick together	Yes	No. Kinetic energy is converted to other forms of energy
Elastic Collision	Objects bounce off each other	Yes	Yes
Explosion	Object or objects break apart	Yes	No. Release of potential energy increases kinetic energy

28 In the absence of external forces, momentum is conserved in which of the following situations?

- A Elastic collisions only.
- B Inelastic collisions only.
- C Explosions only.
- D Elastic and Inelastic collisions and explosions.
- I need help

29 Kinetic Energy is conserved in which of the following situations (assume no external forces)?

- A Elastic collisions only.
- B Inelastic collisions only.
- C Explosions only.
- D Elastic and Inelastic collisions and explosions.
- I need help

30 In an inelastic collision, kinetic energy can be transformed into what after the collision? **Select two answers.**

A Nothing, kinetic energy is conserved.

B More kinetic energy.

C Thermal energy.

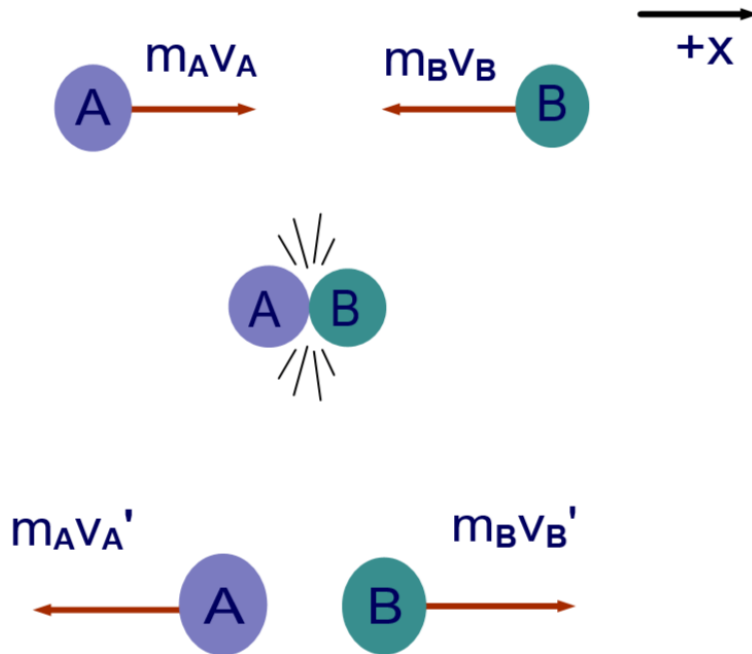
D Light energy.

E I need help



# Conservation of Momentum

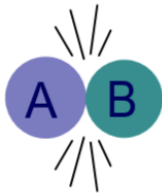
During a collision or an explosion, *measurements show* that the total momentum of a closed system does not change. The diagram below shows the objects approaching, colliding and then separating.



*If the measurements don't show that the momentum is conserved, then this would not be a valid law. Fortunately they do, and it is!*

# Conservation of Momentum

We know velocity is a vector, so to save notation, we'll be leaving off the arrows. The "prime" sign will denote the velocity after the collision or explosion.



$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$



# Perfect Inelastic Collisions

In a perfect inelastic collision, two objects collide and stick together, moving afterwards as one object.

Before (moving towards each other)



After (moving together)



$$p_A + p_B = p'_A + p'_B$$

$$m_A v_A + m_B v_B = (m_A + m_B) v'$$

31 A 13,500 kg railroad freight car travels on a level track at a speed of 4.5 m/s. It collides and couples with a 25,000 kg second car, initially at rest and with brakes released. No external force acts on the system. What is the speed of the two cars after the collision?

0.40 m/s

0.80 m/s

1.6 m/s

3.2 m/s

I need help

32 A cannon ball with a mass of 100.0 kg flies in horizontal direction with a speed of 250 m/s and strikes a ship initially at rest. The mass of the ship is 15,000 kg. Find the speed of the ship after the ball becomes embedded in it.

- 1.3 m/s
- 1.7 m/s
- 2.0 m/s
- 2.4 m/s
- I need help

33 A 40 kg girl skates at 5.5 m/s on ice toward her 70 kg friend who is standing still, with open arms. As they collide and hold each other, what is their speed after the collision?

- A 1.0 m/s
- B 2.0 m/s
- C 4.0 m/s
- D 5.0 m/s
- E I need help

# Explosions

In an explosion, one object breaks apart into two or more pieces (or coupled objects break apart), then moving as separate objects.

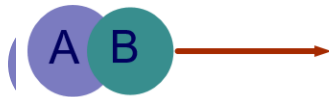
To make the problems a little easier, we will assume:

- the object (or a coupled pair of objects) breaks into two pieces.
- the explosion is along the same line as the initial velocity (two dimensional motion will be covered later in this unit).

# Explosions

Here's a schematic of an explosion. Can you see how it's the time reversed picture of a perfectly inelastic collision?

Before (moving together)



After (moving apart)



$$p_A + p_B = p'_A + p'_B$$

$$(m_A + m_B)v = m_A v'_A + m_B v'_B$$



34 A 5 kg cannon ball is loaded into a 300 kg cannon. When the cannon is fired, it recoils at  $-5$  m/s. What is the cannon ball's velocity after the explosion?

- 60 m/s
- 90 m/s
- 180 m/s
- 300 m/s
- I need help

35 Two railcars, one with a mass of 4000 kg and the other with a mass of 6000 kg, are at rest and stuck together. To separate them a small explosive is set off between them. The 4000 kg car is measured travelling at 6 m/s. How fast is the 6000 kg car going?

-10 m/s

-4 m/s

+4 m/s

+10 m/s

I need help

# Elastic Collisions

In an elastic collision, two objects collide and bounce off each other, as shown below, and both momentum and kinetic energy are conserved.

This will give us two simultaneous equations to solve to predict their motion after the collision.

Before (moving towards)



After (moving apart)



# Elastic Collisions

Before (moving towards)



After (moving apart)



Conservation of Momentum

$$p_A + p_B = p'_A + p'_B$$

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

Conservation of Kinetic Energy

$$KE_A + KE_B = KE'_A + KE'_B$$

$$\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2 = \frac{1}{2} m_A v'^2_A + \frac{1}{2} m_B v'^2_B$$

# Elastic Collisions

Since both of these equations describe the motion of the same system, they must both be true - hence they can be solved as simultaneous equations. Stand by for some algebra.

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

*Note how a minor notation change was made - textbooks and websites use numbers or letters to denote different objects - either work.*

# Elastic Collision Simultaneous Equations

Conservation of Momentum

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$m_1 v_1 - m_1 v_1' = m_2 v_2' - m_2 v_2$$

$$m_1 (v_1 - v_1') = m_2 (v_2' - v_2)$$

Conservation of Kinetic Energy

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2$$

$$m_1 v_1^2 + m_2 v_2^2 = m_1 v_1'^2 + m_2 v_2'^2$$

$$m_1 v_1^2 - m_1 v_1'^2 = m_2 v_2'^2 - m_2 v_2^2$$

$$m_1 (v_1^2 - v_1'^2) = m_2 (v_2'^2 - v_2^2)$$



Does this look familiar?

# Elastic Collision Simultaneous Equations

Conservation of Momentum

$$m_1(v_1 - v_1') = m_2(v_2' - v_2)$$

Conservation of Kinetic Energy

$$m_1(v_1^2 - v_1'^2) = m_2(v_2'^2 - v_2^2)$$

This is the in the same form as the easily factorable expression:

$$a^2 - b^2 = (a + b)(a - b)$$

$$m_1(v_1 + v_1')(v_1 - v_1') = m_2(v_2' + v_2)(v_2' - v_2)$$

The next step is to divide the Conservation of Kinetic Energy by the Conservation of Momentum Equation. You can check with your math teacher - this is ok.

# Elastic Collision Simultaneous Equations

$$\frac{\text{Conservation of Kinetic Energy}}{\text{Conservation of Momentum}} = \frac{m_1 (v_1 + v_1')(v_1 - v_1') = m_2 (v_2' + v_2)(v_2' - v_2)}{m_1 (v_1 - v_1') = m_2 (v_2' - v_2)}$$

$$(v_1 + v_1') = (v_2' + v_2)$$

$$v_1 - v_2 = -(v_1' - v_2')$$

This is what we were looking for by solving the Conservation of Momentum and Kinetic Energy equations simultaneously.

*Do you recognize the terms on the left and right of the equation? And, what do they mean?*



# Properties of Elastic Collisions

$$v_1 - v_2 = -(v'_1 - v'_2)$$

The terms are the relative velocities of the two objects before and after the collision.

It means that for all elastic collisions - independent of each object's mass - the magnitude of the relative velocity of the objects is the same before and after the collision.

36 Two objects have an elastic collision. Before they collide, they are approaching with a velocity of 4 m/s relative to each other. With what velocity do they move apart from one another after the collision?

0 m/s

1 m/s

2 m/s

4 m/s

I need help

37 Two objects have an elastic collision. Object  $m_1$ , has an initial velocity of +4 m/s and  $m_2$  has a velocity of -3 m/s. After the collision,  $m_1$  has a velocity of 1 m/s. What is the velocity of  $m_2$ ?

-6 m/s

-1 m/s

4 m/s

8 m/s

I need help

38 Two objects have an elastic collision. Object  $m_1$  is to the left of  $m_2$  with an initial velocity of +6 m/s and  $m_2$  has a velocity of +2 m/s. After the collision,  $m_1$  has a velocity of +1 m/s. What is the velocity of  $m_2$ ?

- A 2 m/s
- B 3 m/s
- C 5 m/s
- D 6 m/s
- E I need help

# Properties of Elastic Collisions

Using the Conservation of Kinetic energy and the Conservation of Momentum equations, and solving them in a slightly different manner (the algebra is a little more intense, but not too bad, and might be fun to try), the following equations result:

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

Nice symmetry - take a moment to compare the two equations.

# Properties of Elastic Collisions

Using these equations, we will predict the motion of three specific cases, and relate them to observation:

- Two equal mass particles collide.
- A heavy particle collides with a light particle at rest.
- A light particle collides with a heavy particle at rest.

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

# Collision of same mass particles

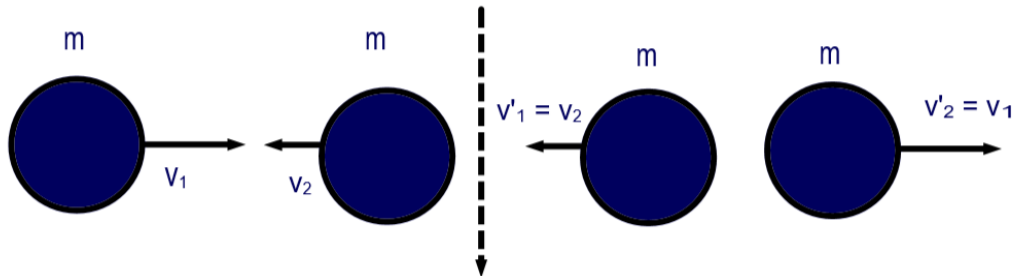
$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

For a collision of two particles of the same mass,  $m = m_1 = m_2$ . When this substitution is made into the two equations on the left (try it!), they simplify to:

$$v_1' = v_2$$

$$v_2' = v_1$$

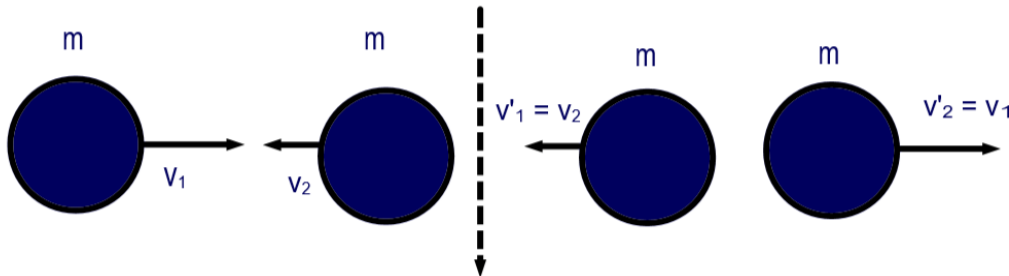


# Collision of same mass particles

$$v'_1 = v_2$$
$$v'_2 = v_1$$

The particles exchange velocities. A good example of this is playing billiards. The cue ball and the striped and solid balls are all the same mass.

When the cue ball hits a solid ball, the cue ball stops, and the solid ball takes off with the cue ball's velocity (ignoring friction and the rotational motion of the pool balls).





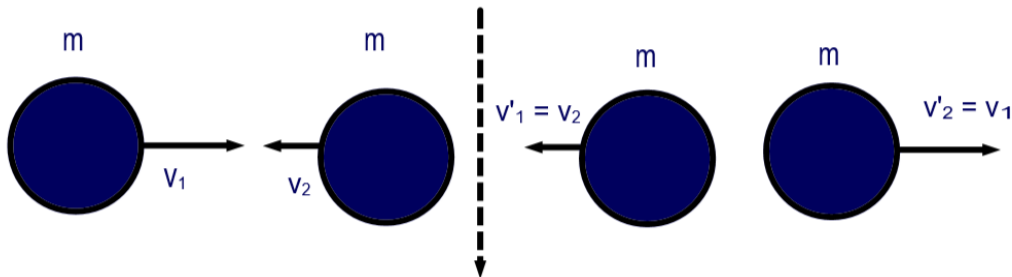
# Collision of same mass particles

$$v'_1 = v_2$$

$$v'_2 = v_1$$

Another good example is found in nuclear reactor engineering. When Uranium 235 fissions, it releases 3 high speed neutrons.

These neutrons are moving too fast to cause another Uranium 235 nucleus to fission, so they have to be slowed down by colliding with another atom or molecule. What type would be of most use here?



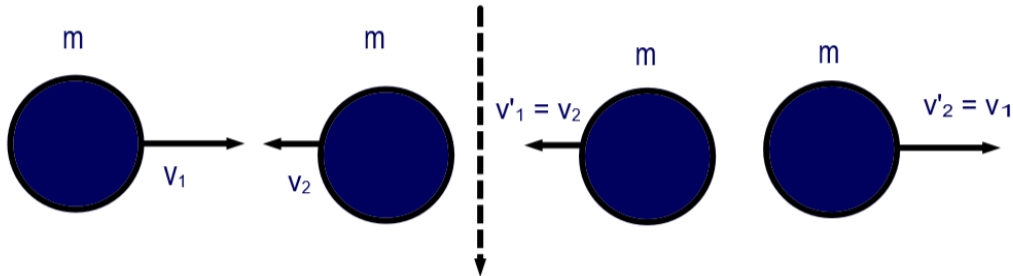
# Collision of same mass particles

$$v'_1 = v_2$$

$$v'_2 = v_1$$

An atom with a similar mass to neutrons. A Hydrogen atom moving slower than the neutrons would be great - the neutron would exchange velocities, slowing down, while speeding up the hydrogen atom.

Operational reactors use water for this purpose (heavier than hydrogen, but has many other practical benefits - nuclear engineering is not as simple as analyzing elastic collisions!).



# Large mass striking lighter mass at rest

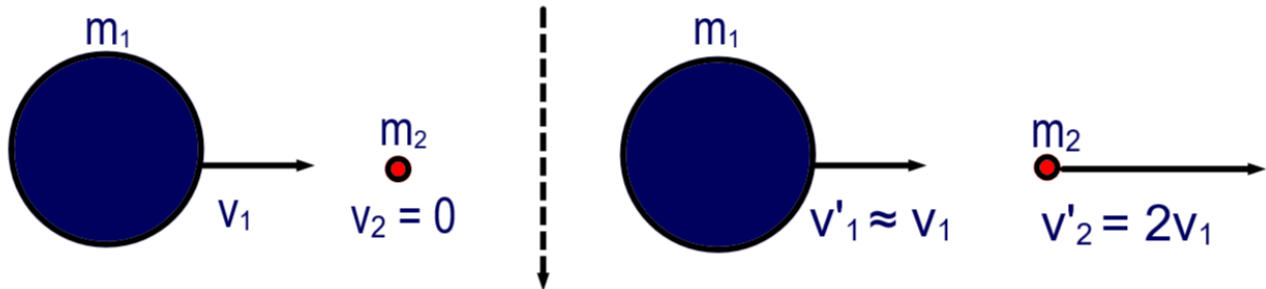
$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$

$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

For this case,  $m_1 \gg m_2$  ( $\gg$  is the symbol for much greater), which means that  $m_1 \pm m_2 \approx m_1$ . The equations simplify to:

$$v_1' = v_1$$

$$v_2' = 2v_1$$

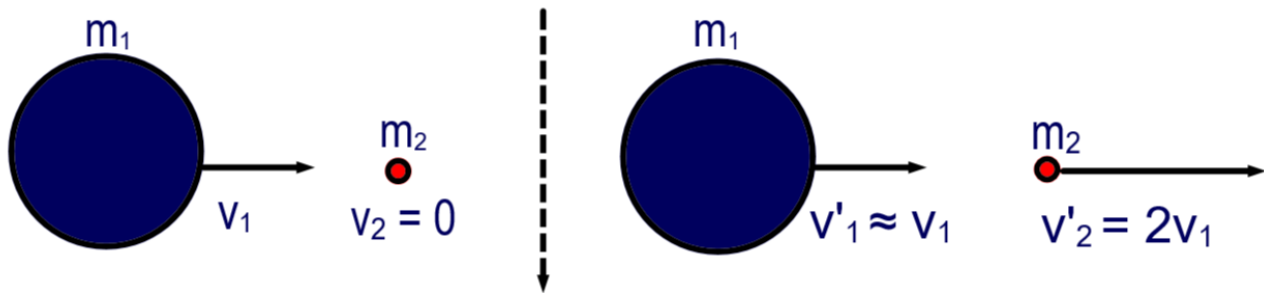


# Large mass striking lighter mass at rest

$$v'_1 = v_1$$

$$v'_2 = 2v_1$$

The large mass particle continues on its way with a slightly smaller velocity. The small particle moves at twice the initial speed of the large mass particle. Picture a bowled bowling ball striking a beach ball at rest.



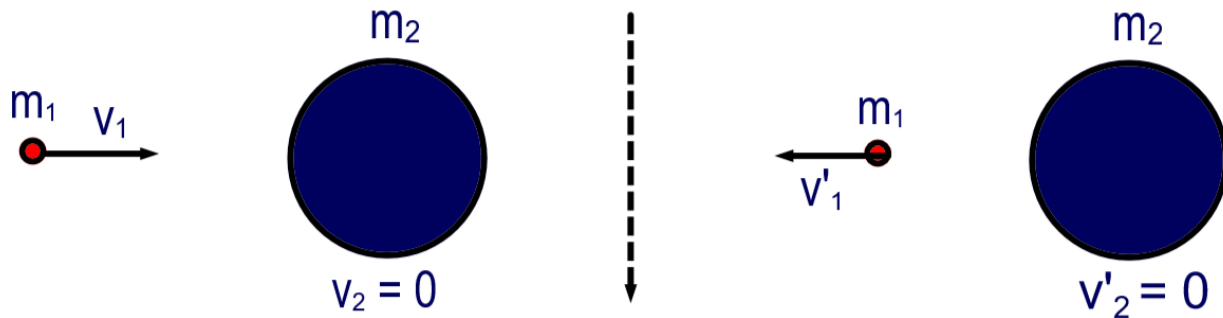
# Small mass striking heavier mass at rest

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1 + \frac{2m_2}{m_1 + m_2} v_2$$
$$v_2' = \frac{2m_1}{m_1 + m_2} v_1 + \frac{m_2 - m_1}{m_1 + m_2} v_2$$

For this case,  $m_1 \ll m_2$  ( $\ll$  is the symbol for much less than), which means that  $m_1 \pm m_2 \approx m_2$ . The equations simplify to:

$$v_1' = -v_1$$

$$v_2' = 0$$

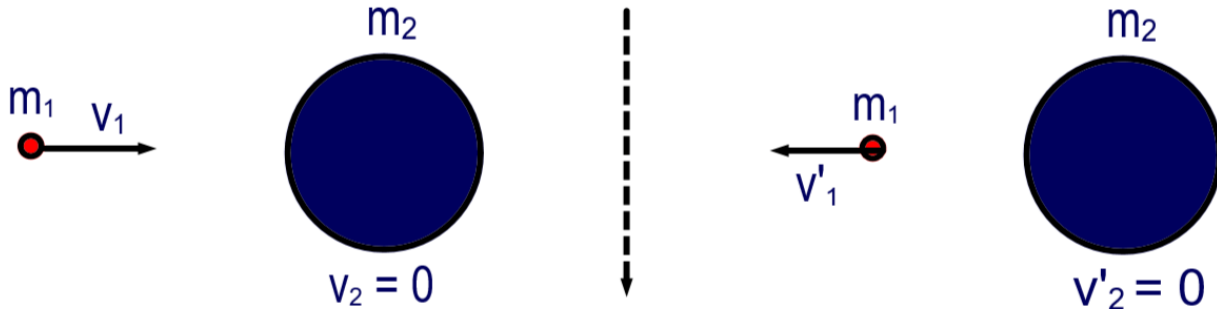


# Small mass striking heavier mass at rest

$$v_1' = -v_1$$

$$v_2' = 0$$

The large mass particle remains mostly at rest. The small particle reverses direction, with the same magnitude as its initial velocity. Picture a moving golf ball hitting a stationary bowling ball.



39 A bowling ball, with a velocity of  $+v$ , collides with a ping pong ball at rest. The velocity of the bowling ball is virtually unaffected by the collision. What will be the velocity of the ping pong ball?

- A  $-2v$
- B  $-v$
- C  $+v$
- D  $+2v$
- I need help

40 A baseball bat has a velocity of  $+v$  when it elastically collides with a baseball that has a velocity of  $-2v$ . The bat barely changes velocity during the collision. How fast is the baseball going after it's hit?

3 v

4 v

7 v

8 v

I need help



41 Two objects with identical masses have an elastic collision: the initial velocity of  $m_1$  is +6 m/s and  $m_2$  is -3 m/s. What is the velocity of  $m_1$  after the collision?

- A -6 m/s
- B -3 m/s
- C 3 m/s
- D 6 m/s
- E I need help

42 Two objects with identical masses have an elastic collision: the initial velocity of  $m_1$  is +6 m/s and  $m_2$  is -3 m/s. What is the velocity of  $m_2$  after the collision?

- A -6 m/s
- B -3 m/s
- C 3 m/s
- D 6 m/s
  
- I need help

43 A golf ball is hit against a solid cement wall, and experiences an elastic collision. The golf ball strikes the wall with a velocity of  $+35$  m/s. What velocity does it rebound with?

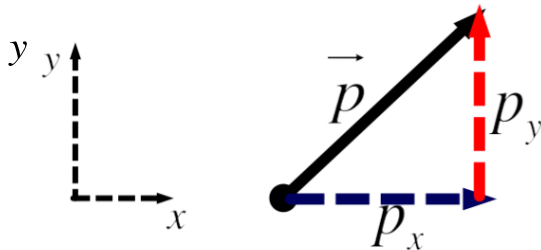
- A  $-70$  m/s
- B  $-35$  m/s
- C  $35$  m/s
- D  $70$  m/s
  
- I need help

# Collisions in Two Dimensions

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# Conservation of Momentum in Two Dimensions

Momentum vectors (like all vectors) can be expressed in terms of component vectors relative to a reference frame



*This, of course also applies to three dimensions, but we'll stick with two for this chapter!*

This means that the momentum conservation equation  $p = p'$  can be solved independently for each component:

$$p_x = p'_x$$

$$p_y = p'_y$$

# General Two Dimensional Collisions

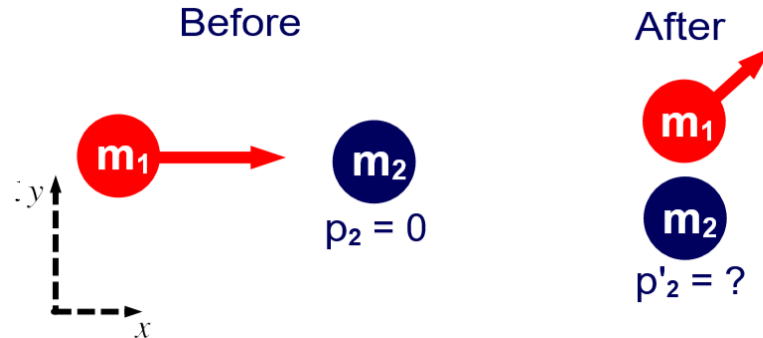


Consider a system of two objects moving in random directions in a two dimensional plane and colliding.

Assume there are no external forces acting on the system.

Since there is no absolute reference frame, we'll line up the x-axis with the velocity of one of the objects.

# General Two Dimensional Collisions

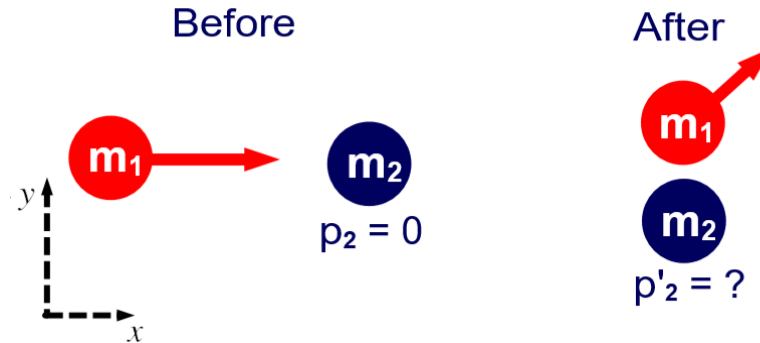


*This is not a head on collision - note how  $m_1$  heads off with a  $y$  component of velocity after it strikes  $m_2$ . Also, did you see how we rotated the coordinate system so the  $x$  axis is horizontal?*

*To work a simpler problem, we'll assume that mass 2 is at rest.*

Find the momentum of  $m_2$  after the collision.

# General Two Dimensional Collisions



Look at the vectors first - momentum must be conserved in both the x and the y directions.

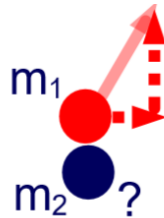
Since the system momentum in the y direction is zero before the collision, it must be zero after the collision.

The value that  $m_1$  has for momentum in the x direction must be shared between both objects after the collision - *and not equally* - it will depend on the masses and the separation angle.

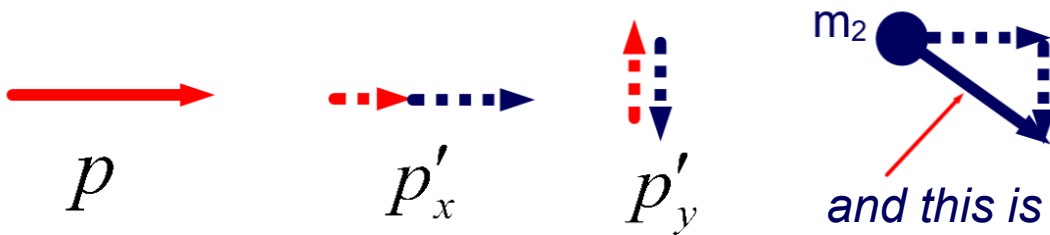


# General Two Dimensional Collisions

Here is the momentum vector breakdown of mass 1 after the collision:

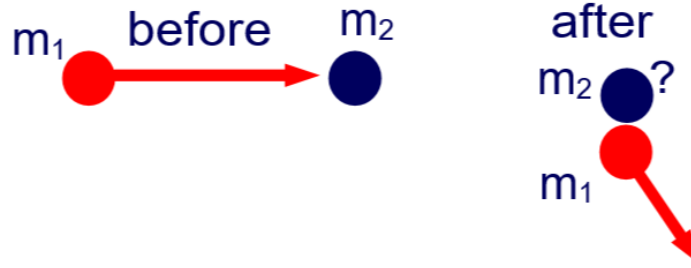







$m_2$  needs to have a component in the y direction to sum to zero with  $m_1$ 's final y momentum. And it needs a component in the x direction to add to  $m_1$ 's final x momentum to equal the initial x momentum of  $m_1$ :



*and this is the final momentum for mass 2 by vectorially adding the final  $p_x$  and  $p_y$ .*

44 After the collision shown below, which of the following is the most likely momentum vector for the blue ball?

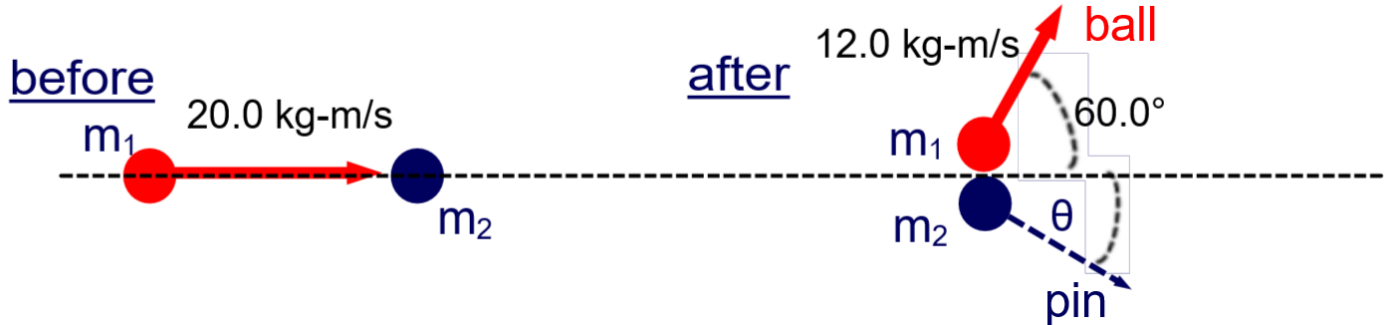


- A 
- B 
- C 
- D 
- E 
- F I need help

Answer

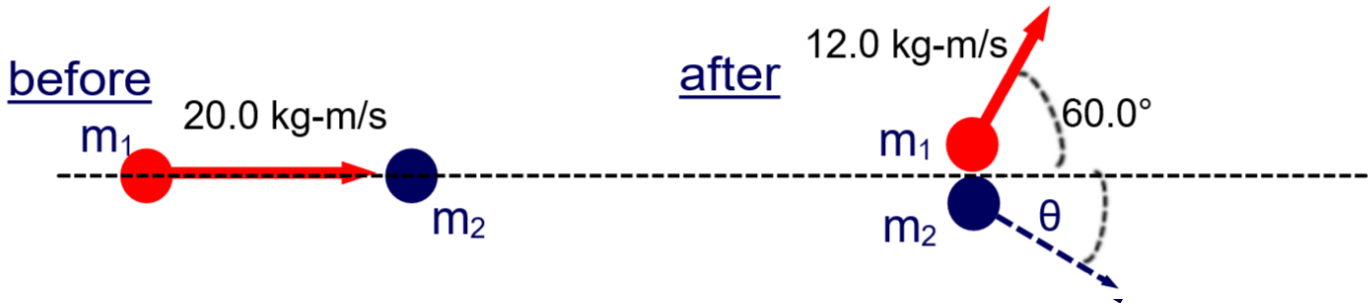
# General Two Dimensional Collisions

Now that we've seen the vector analysis, let's run through the algebra to find the momentum (magnitude and direction) that  $m_2$  leaves with after the collision.



There is a bowling ball ( $m_1$ ) with momentum  $20.0 \text{ kg}\cdot\text{m/s}$  that strikes a stationary bowling pin ( $m_2$ ) and then the bowling ball and pin take off as shown above. What is the final momentum of the pin?

# General Two Dimensional Collisions



Given:

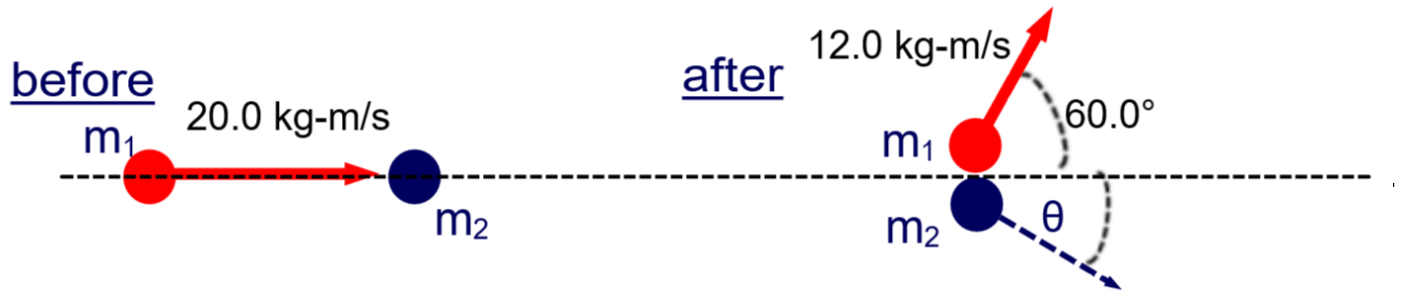
$$p_{1x} = 20 \text{ kg m/s} \quad p_{2x} = 0$$
$$p_{1y} = 0 \quad p_{2y} = 0$$

$$p'_{1x} = 12 \cos 60 \text{ kg m/s}$$
$$p'_{1y} = 12 \sin 60 \text{ kg m/s}$$

Find:

$$p'_2 = \sqrt{(p'_{2x})^2 + (p'_{2y})^2}$$
$$\theta = \tan^{-1} \frac{p'_{2y}}{p'_{2x}}$$

# General Two Dimensional Collisions



Use Conservation of Momentum in the x and y directions.

x direction

$$p_{1x} + p_{2x} = p'_{1x} + p'_{2x}$$

$$p'_{2x} = p_{1x} + p_{2x} - p'_{1x}$$

$$p'_{2x} = (20 + 0 - 12 \cos 60) \text{ kg m/s}$$

$$p'_{2x} = 14.0 \text{ kg m/s}$$

y-direction

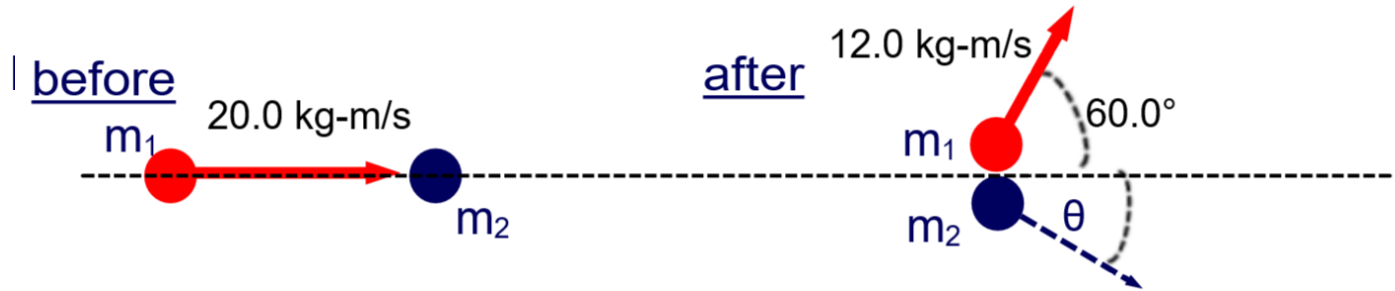
$$p_{1y} + p_{2y} = p'_{1y} + p'_{2y}$$

$$p'_{2y} = p_{1y} + p_{2y} - p'_{1y}$$

$$p'_{2y} = (0 + 0 - 12 \sin 60) \text{ kg m/s}$$

$$p'_{2y} = -10.4 \text{ kg m/s}$$

# General Two Dimensional Collisions



Now that the x and y components of the momentum of mass 2 have been found, the final momentum of the bowling pin is calculated.

$$p'_{2x} = 14.0 \text{ kg m/s}$$

$$p'_{2y} = -10.4 \text{ kg m/s}$$

$$p'_2 = \sqrt{(p'_{2x})^2 + (p'_{2y})^2}$$

$$p'_2 = \sqrt{(14.0 \text{ kg m/s})^2 + (-10.4 \text{ kg m/s})^2}$$

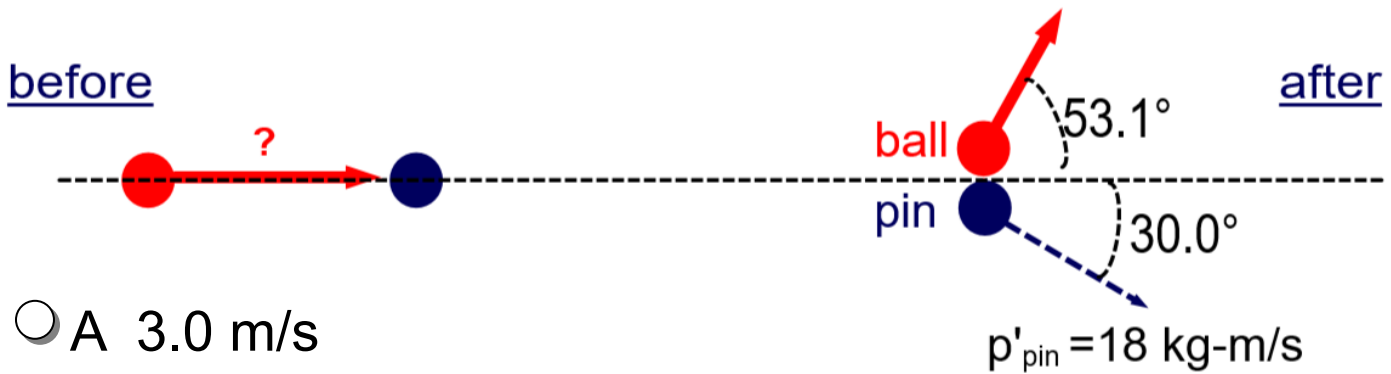
$$p'_2 = 17.4 \text{ kg m/s}$$

$$\theta = \tan^{-1} \frac{p'_{2y}}{p'_{2x}}$$

$$\theta = \tan^{-1} \frac{-10.4 \text{ kg m/s}}{14.0 \text{ kg m/s}}$$

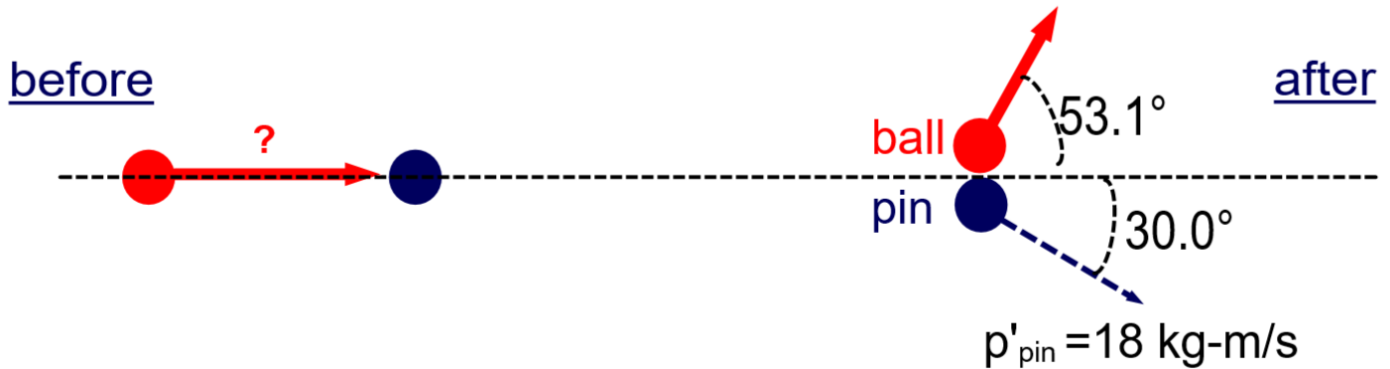
$$\theta = -36.6^\circ$$

45 A 5.0 kg bowling ball strikes a stationary bowling pin. After the collision, the ball and the pin move in directions as shown and the magnitude of the pin's momentum is 18 kg-m/s. What was the velocity of the ball before the collision?



- A 3.0 m/s
- B 4.5 m/s
- C 5.0 m/s
- D 6.0 m/s
- E I need help

# Two Dimensional Collisions Problem Solution



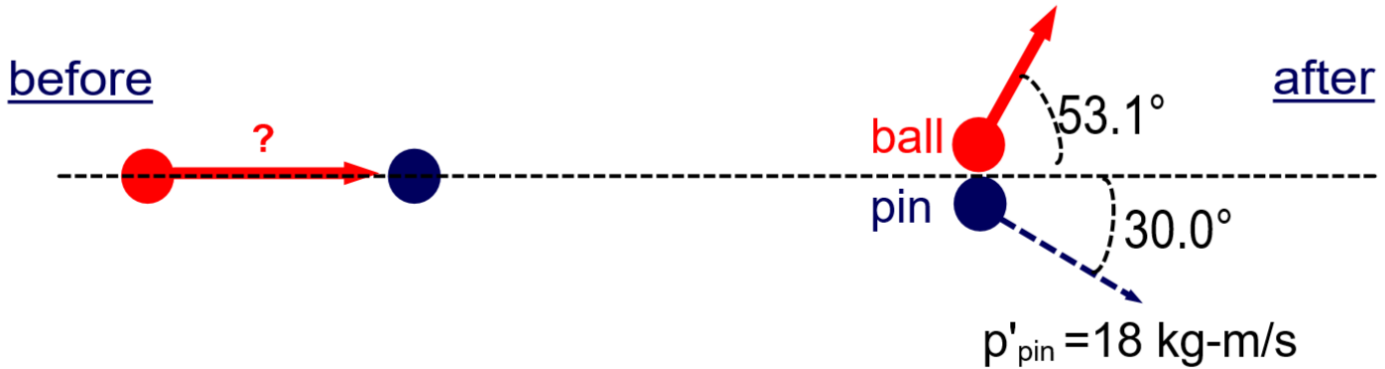
Start with the given quantities - and note that this problem will be worked backwards; the final, not the initial conditions are specified.

Given:  $\theta'_{ball} = \theta'_1 = 53.1^\circ$   
 $\theta'_{pin} = \theta'_2 = 30.0^\circ$   
 $p'_2 = 18 \text{ kg}\cdot\text{m/s}$

Find:  $v_{ball} = v_1$



# Two Dimensional Collisions Problem Solution



Conservation of Momentum in x and y directions:

$$p_{1x} + p_{2x} = p'_{1x} + p'_{2x}$$

$$p_{1x} = p'_{1x} + p'_{2x}$$

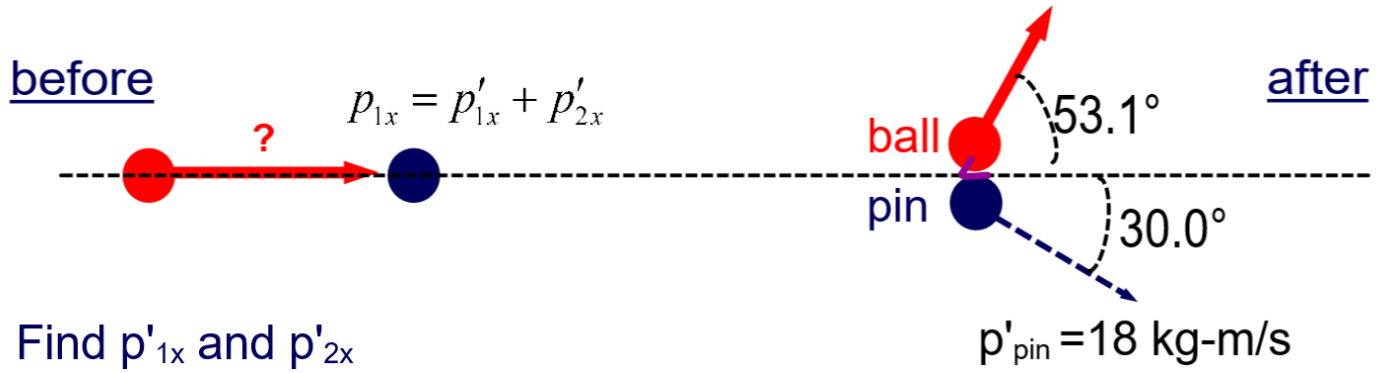
$$p_{1y} + p_{2y} = p'_{1y} + p'_{2y}$$

$$0 + 0 = p'_{1y} + p'_{2y}$$

$$p'_{1y} = -p'_{2y}$$

To find the initial velocity of the bowling ball, we need to find  $p_{1x}$ , and then divide it by the mass of the ball.

# Two Dimensional Collisions Problem Solution



$$\frac{p'_{1y}}{p'_{1x}} = \tan(53.1^\circ)$$

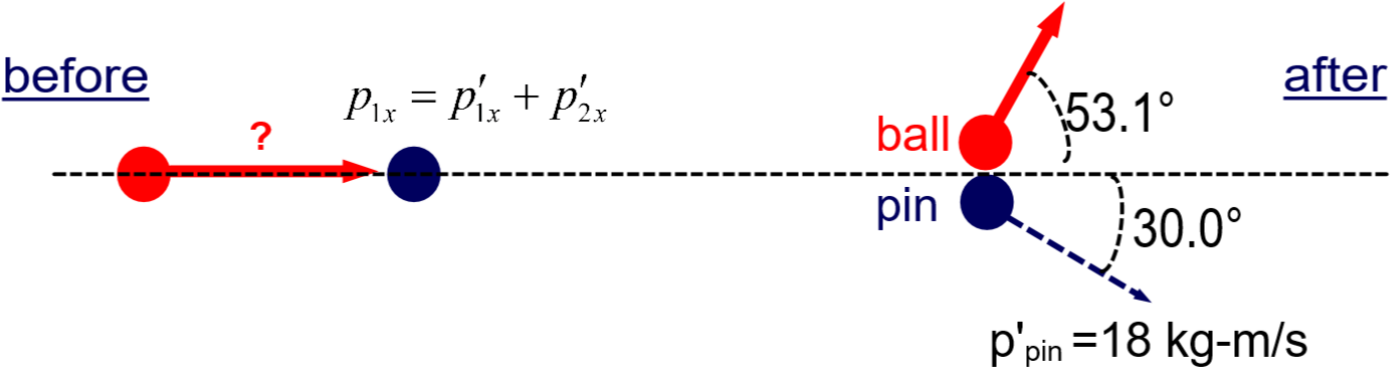
$$p'_{1x} = 0.751 p'_{1y}$$

$$p'_{2x} = 18 \cos 30^\circ$$

$$p'_{2x} = 15.6$$

We now have to find  $p'_{1y}$  in order to find  $p'_{1x}$

# Two Dimensional Collisions Problem Solution



$$p'_{1y} = -p'_{2y}$$

$$p'_{1y} = -18 \sin(-30^\circ) = 9$$

$$p'_{1x} = 0.751 p'_{1y} = 6.76$$

$$p_1 = p_{1x} = p'_{1x} + p'_{2x} = 15.6$$

$$p_1 = 6.76 + 15.6 = 22.4 \text{ kg m/s}$$

$$v_1 = \frac{p_1}{m_1}$$

$$v_1 = \frac{22.4 \text{ kg m/s}}{5.0 \text{ kg}}$$

$$v_1 = 4.5 \text{ m/s}$$

Final answer!

# Perfect Inelastic Collisions in Two Dimensions

A common perfect inelastic collision is where two cars collide and stick at an intersection.

The cars are traveling along paths that are perpendicular just prior to the collision.



# Perfect Inelastic Collisions in Two Dimensions



p-conservation in x:  $p_{1x} = p'_x$     in y:  $p_{2y} = p'_y$

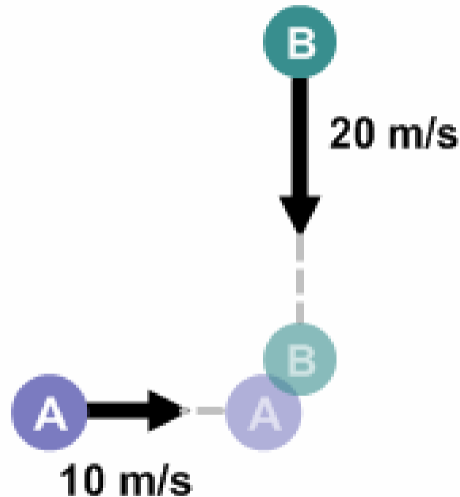
final momentum:  $p' = \sqrt{(p'_x)^2 + (p'_y)^2}$

final velocity:  $v' = p' / (m_1 + m_2)$

final direction:  $\theta = \tan^{-1} \left( \frac{p'_y}{p'_x} \right)$

46 Object A with mass 20.0 kg travels to the east at 10.0 m/s and object B with mass 5.00 kg travels south at 20.0 m/s. They collide and stick together. What is the velocity (magnitude and direction) of the objects after the collision?

- 26.6°
- 34.9°
- 49.2°
- 55.8°
- I need help

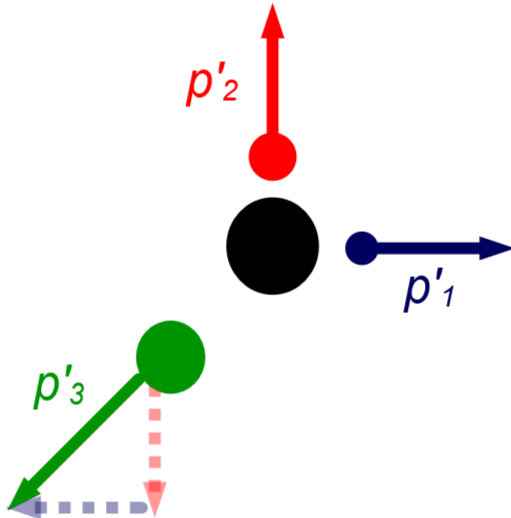


Answer

# Explosions in Two Dimensions

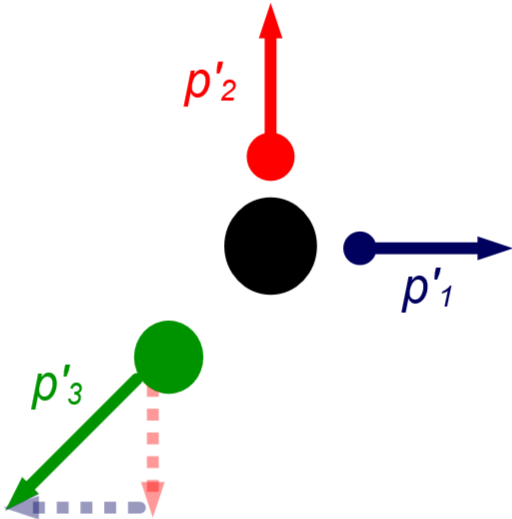
The Black object explodes into 3 pieces (blue, red and green).

We want to determine the momentum of the third piece, given the momentum of the pieces 1 and 2.



*Can you see why the third piece moves the way it is shown?*

# Explosions in Two Dimensions



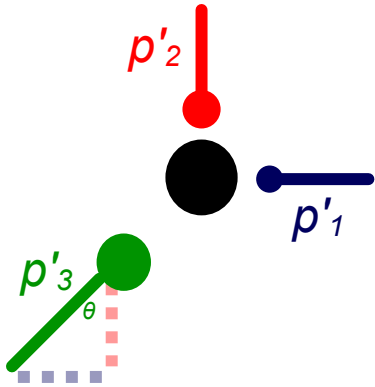
During an explosion, the total momentum is unchanged, since no external force acts on the system.

- By Newton's Third Law, the forces that occur between the particles within the object will add up to zero, so they don't affect the momentum.
- If the initial momentum is zero, the final momentum is zero.
- The third piece must have equal and opposite momentum to the sum of the other two pieces.



# Explosions in Two Dimensions

before:  $p_x = p_y = 0$



The Black object explodes into 3 pieces (blue, red and green). We want to determine the momentum of the third piece.

In this case the blue and red pieces are moving perpendicularly to each other, so:

after:  $p'_{1x} + p'_{2x} + p'_{3x} = 0$

$$p'_{1y} + p'_{2y} + p'_{3y} = 0$$

$$p'_{1y} = 0 \quad \text{---} \quad p'_{3y} = -p'_{2y}$$


$$p'_{2x} = 0 \quad \text{---} \quad p'_{3x} = -p'_{1x}$$

$$p'_3 = \sqrt{(p'_{3x})^2 + (p'_{3y})^2}$$

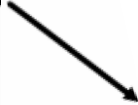
$$p'_3 = \sqrt{(-p'_{1x})^2 + (-p'_{2y})^2}$$


$$\theta = \tan^{-1} \left( \frac{-p'_{2x}}{-p'_{1y}} \right)$$

47 A stationary cannon ball explodes in three pieces. The momenta of two of the pieces is shown below. What is the direction of the momentum of the third piece?

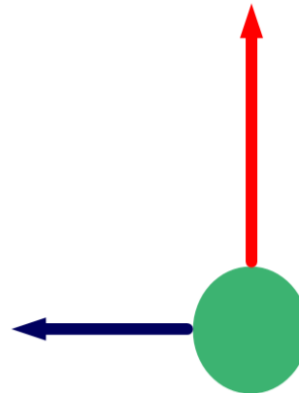
A 

B 

C 

D 

E I need help



Answer

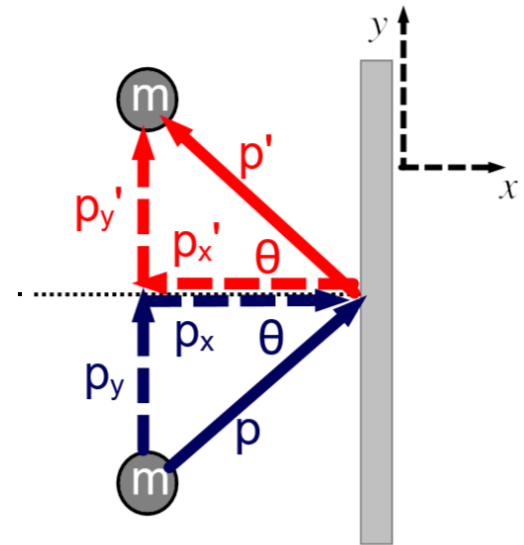
48 A stationary 10.0 kg bomb explodes into three pieces. A 2.00 kg piece moves west at 200.0 m/s. Another piece with a mass of 3.00 kg moves north with a velocity of 100.0 m/s. What is the velocity (speed and direction) of the third piece?

- A  $-22.6^\circ$
- B  $-29.7^\circ$
- C  $-36.9^\circ$
- D  $-44.3^\circ$
- E I need help

# Example: Collision with a Wall

A golf ball collides elastically with a rigid wall that does not move (ignore any impact on the internal energy of the wall). The angle of incidence equals the rebound angle of the ball.

*Given the constraints, what can be said about the momenta in the  $x$  and  $y$  directions?*



The solid lines represent the momentum of the ball (blue - prior to collision, red - after the collision). The dashed lines are the  $x$  and  $y$  components of the momentum vectors.

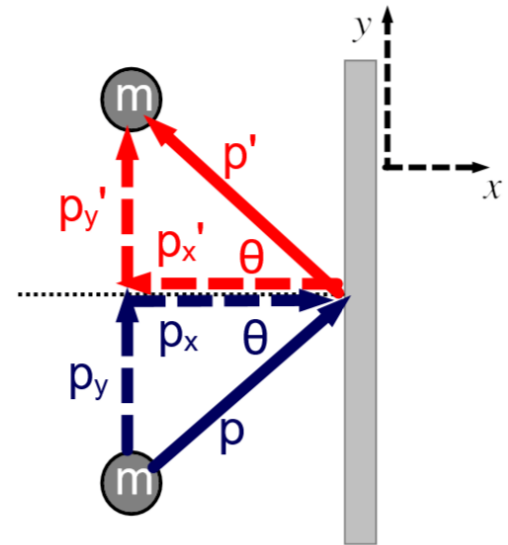
# Example: Collision with a Wall

An external force from the wall is being applied to the ball in order to reverse its direction in the x axis.

Since we're assuming an elastic collision, the ball bounces off the wall with the same speed that it struck the wall with.

Hence, the magnitude of the initial momentum and the final momentum is equal:

$$|\vec{p}'| = |\vec{p}| = p' = p$$



*Now it's time to resolve each momentum into components along the x and y axis.*

# Example: Collision with a Wall

$$p_x = p \cos \theta$$

$$p_y = p \sin \theta$$

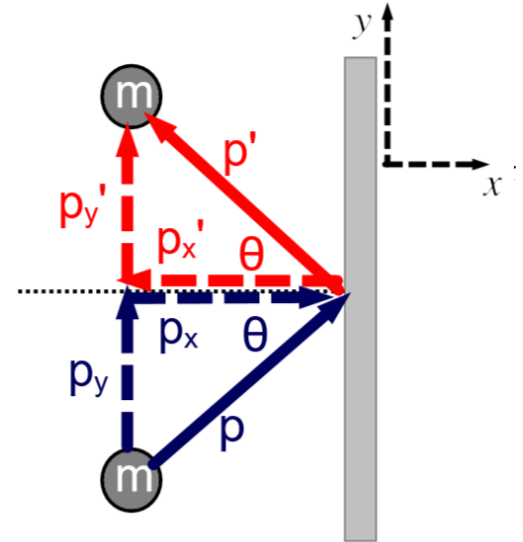
$$p'_x = -p' \cos \theta = -p \cos \theta$$

$$p'_y = p' \sin \theta = p \sin \theta$$

Apply the Impulse Momentum Theorem in two dimensions, where the wall is exerting the external force:

$$F_x \Delta t = p'_x - p_x = -p \cos \theta - (p \cos \theta) = -2p \cos \theta$$

$$F_y \Delta t = p'_y - p_y = p \sin \theta - p \sin \theta = 0$$



*What does this tell us?*

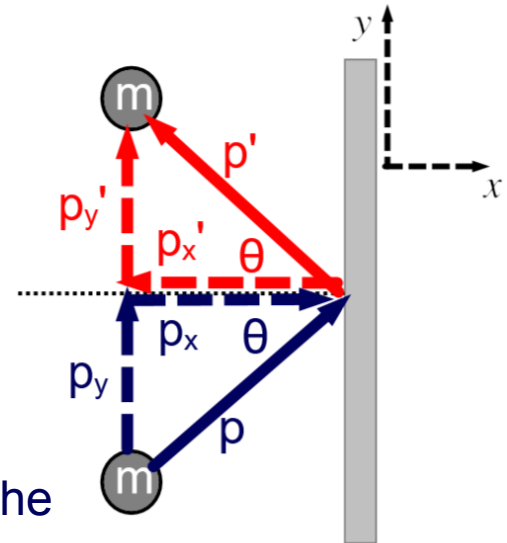
# Example: Collision with a Wall

$$F_x \Delta t = p'_x - p_x = -2p \cos \theta$$

$$F_y \Delta t = p'_y - p_y = 0$$

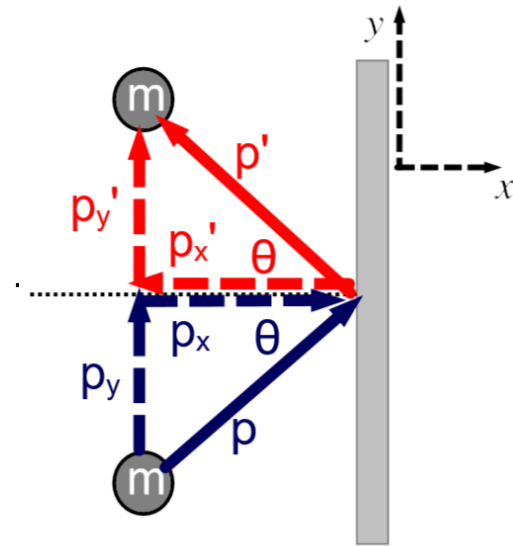
Momentum is conserved in the y direction, but it is not conserved in the x direction - the only force is the force exerted by the wall in the -x direction - the Normal Force.

The initial and final momentum in the y direction are the same and subtract out. The initial and final momentum in the x direction are equal in magnitude, but opposite in direction - and add up to a value in the -x direction.



49 A tennis ball of mass  $m$  strikes a wall at an angle  $\theta$  relative to normal then bounces off with the same speed as it had initially. What is the change in momentum of the ball in the  $x$  direction?

- A 0
- B  $-2mv$
- C  $-mv \cos\theta$
- D  $-2mv \cos\theta$
- I need help

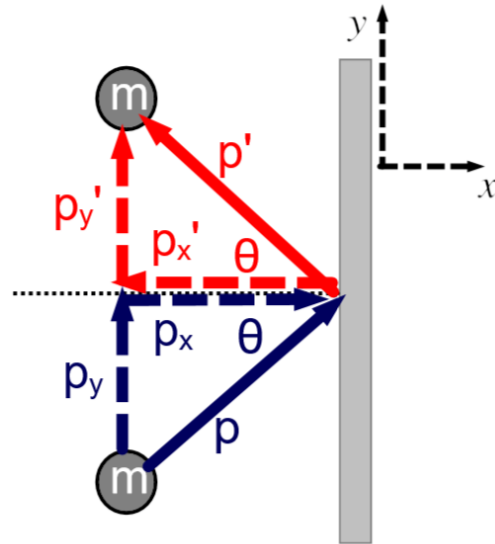


Answer



50 A tennis ball of mass  $m$  strikes a wall at an angle  $\theta$  relative to normal then bounces off with the same speed as it had initially. What is the change in momentum of the ball in the  $y$  direction?

- A 0
- B  $-2mv$
- C  $-mv \cos\theta$
- D  $-2mv \cos\theta$
- I need help



Answer