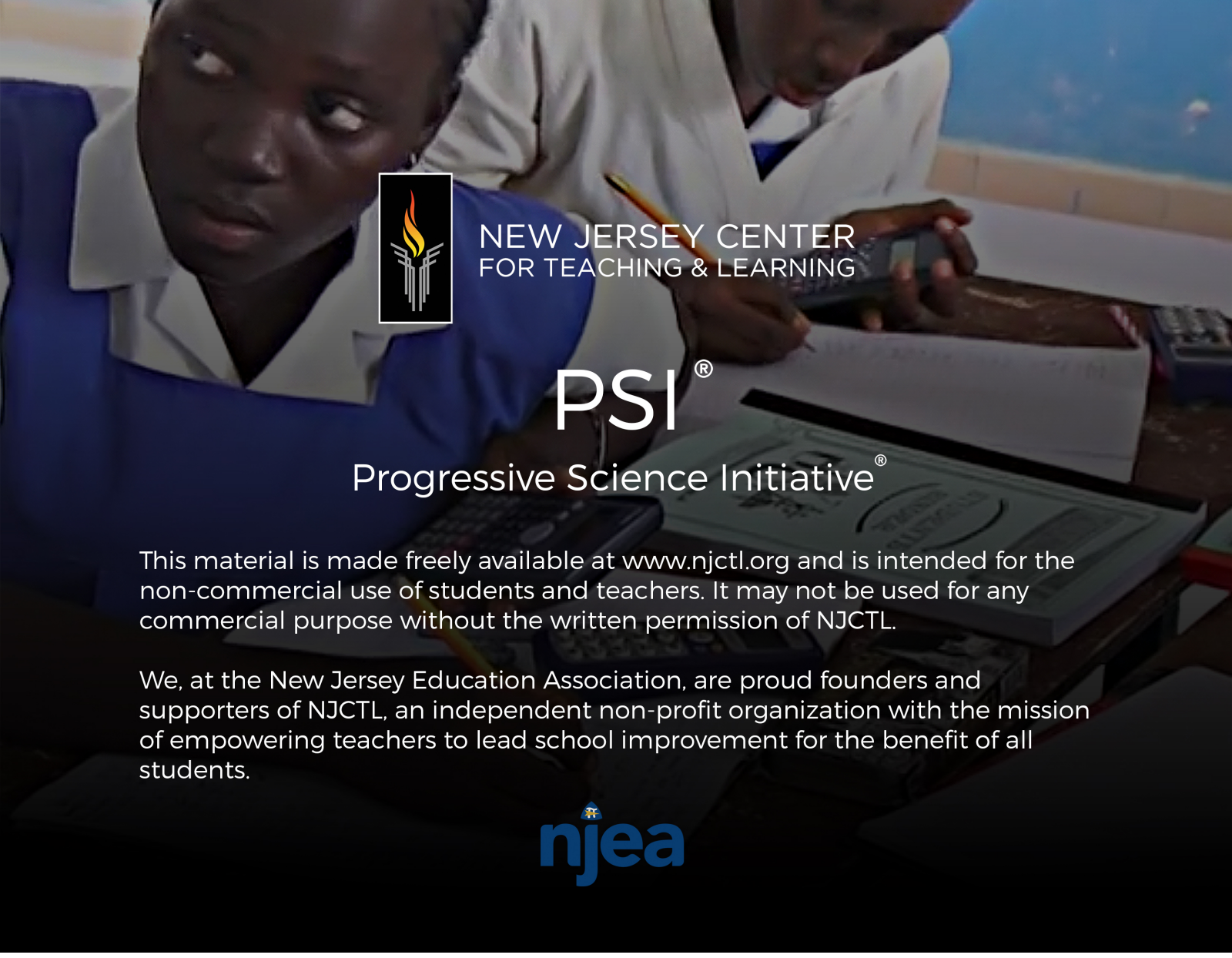

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Algebra Based Physics

Simple Harmonic Motion

2017-07-14

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Click on the topic to go to that section

Period and Frequency

https://www.njctl.org/video/?v=WYPYUR3mK6o

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SHM and Circular Motion

There is a deep connection between Simple Harmonic Motion (SHM) and Uniform Circular Motion (UCM).

Simple Harmonic Motion can be thought of as a one-dimensional projection of Uniform Circular Motion.

All the ideas we learned for UCM, can be applied to SHM...we don't have to reinvent them.

So, let's review circular motion first, and then extend what we know to SHM.

Click here to see how circular motion relates to simple harmonic motion.

Period

The time it takes for an object to complete one trip around a circular path is called its Period.

The symbol for Period is "T"

Periods are measured in units of time; we will usually use seconds (s).

Often we are given the time (t) it takes for an object to make a number of trips (n) around a circular path. In that case,

$$T = \frac{t}{n}$$

1 If it takes 50 seconds for an object to travel around a circle 5 times, what is the period of its motion?

- ○ 5 s
- ○ 10 s
- ○ 25 s
- ○ 40 s
- ○ I need help

Answer

https://www.njctl.org/video/?v=BHryxYo-Tdc

2 If an object is traveling in circular motion and its period is 7.0s, how long will it take it to make 8 complete revolutions?

- A 1.1 s
- B 11 s
- C 39 s
- D 56 s
- E I need help

Answer

https://www.njctl.org/video/?v=EEfrv73yUho

Frequency

The number of revolutions that an object completes in a given amount of time is called the frequency of its motion.

The symbol for frequency is "f"

Periods are measured in units of revolutions per unit time; we will usually use 1/seconds (s^{-1}). Another name for s^{-1} is Hertz (Hz). Frequency can also be measured in revolutions per minute (rpm), etc.

Often we are given the time (t) it takes for an object to make a number of revolutions (n). In that case,

$$f = \frac{n}{t}$$

https://www.njctl.org/video/?v=ct9ENPVZjUY

3 An object travels around a circle 50 times in ten seconds, what is the frequency (in Hz) of its motion?

- A 5.0 Hz
- B 10 Hz
- C 50 Hz
- D 500 Hz
- E I need help

Answer

https://www.njctl.org/video/?v=T52fxgs1qg0

4 If an object is traveling in circular motion with a frequency of 7.0 Hz, how many revolutions will it make in 20s?

- 3.0
- 13
- 38
- 140
- I need help

Answer

https://www.njctl.org/video/?v=uxu9q_WGcjc

Period and Frequency

Since $T = \frac{t}{n}$ and $f = \frac{n}{t}$

then $T = \frac{1}{f}$ and $f = \frac{1}{T}$

https://www.njctl.org/video/?v=BLxKNxaN0vA

5 An object has a period of 4.0s, what is the frequency of its motion (in Hertz)?

- A 0.25 Hz
- B 0.41 Hz
- C 0.67 Hz
- D 0.89 Hz
- E I need help

Answer

https://www.njctl.org/video/?v=eAkyGQSkZwk

6 An object is revolving with a frequency of 8.0 Hz, what is its period (in seconds)?

- 0.020 s
- 0.070 s
- 0.11 s
- 0.13 s
- I need help

Answer

Velocity

Also, recall from Uniform Circular Motion....

$$v = \frac{2\pi r}{T}$$

and

$$v = 2\pi r f$$

https://www.njctl.org/video/?v=EYm9VH6TIas

7 An object is in circular motion. The radius of its motion is 2.0 m and its period is 5.0s. What is its velocity?

- A 1.9 m/s
- B 2.5 m/s
- C 3.1 m/s
- D 4.6 m/s
- E I need help

Answer

https://www.njctl.org/video/?v=waqsDkFEDiY

8 An object is in circular motion. The radius of its motion is 2.0 m and its frequency is 8.0 Hz. What is its velocity?

- A 14 m/s
- B 34 m/s
- C 67 m/s
- D 105 m/s
- E I need help

Answer

https://www.njctl.org/video/?v=pGQClKclL9E

SHM and UCM

https://www.njctl.org/video/?v=FaYmq0h1Fa4

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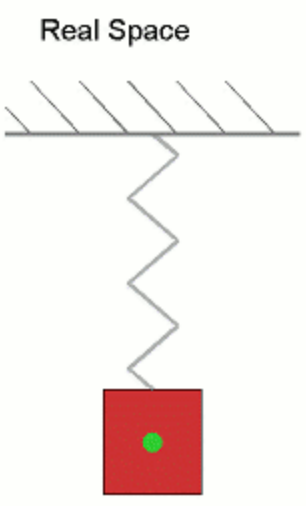
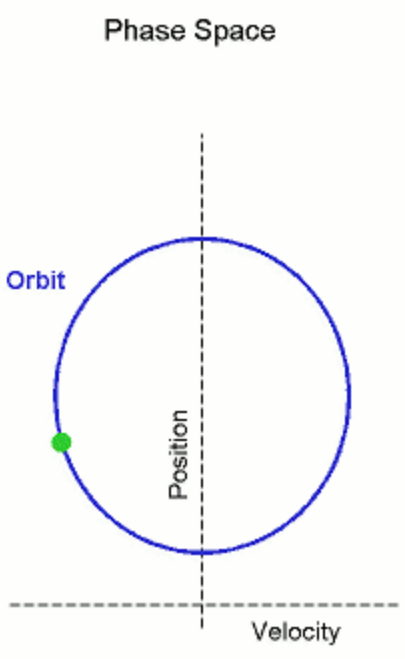
SHM and Circular Motion

In UCM, an object completes one circle, or cycle, in every T seconds. That means it returns to its starting position after T seconds.

In Simple Harmonic Motion, the object does not go in a circle, but it also returns to its starting position in T seconds.





Any motion that repeats over and over again, always returning to the same position is called " periodic".

Click here to see how simple harmonic motion relates to circular motion.

9 It takes 4.0s for a system to complete one cycleof simple harmonic motion. What is thefrequency of the system?

- 0.25 Hz
- 0.41 Hz
- 0.56 Hz
- 0.83 Hz
- I need help

Answer

https://www.njctl.org/video/?v=2XjMq2RCHHk

10 The period of a mass-spring system is 4.0s and the amplitude of its motion is 0.50m. How far does the mass travel in 4.0s?

- 1.0 m
- 2.0m
- 4.0 m
- 5.0 m
- I need help

Answer

https://www.njctl.org/video/?v=PhUf12Yhtok

11 The period of a mass-spring system is 4.0s and the amplitude of its motion is 0.50m. How far does the mass travel in 6.0s?

- ❍ 3.0 m
- ❍ 6.0 m
- ❍ 9.0 m
- ❍ 12 m
- ❍ I need help

Answer

https://www.njctl.org/video/?v=2CBsSb3rjZk

Spring Pendulum

https://www.njctl.org/video/?v=-CO5LHBWb4Y

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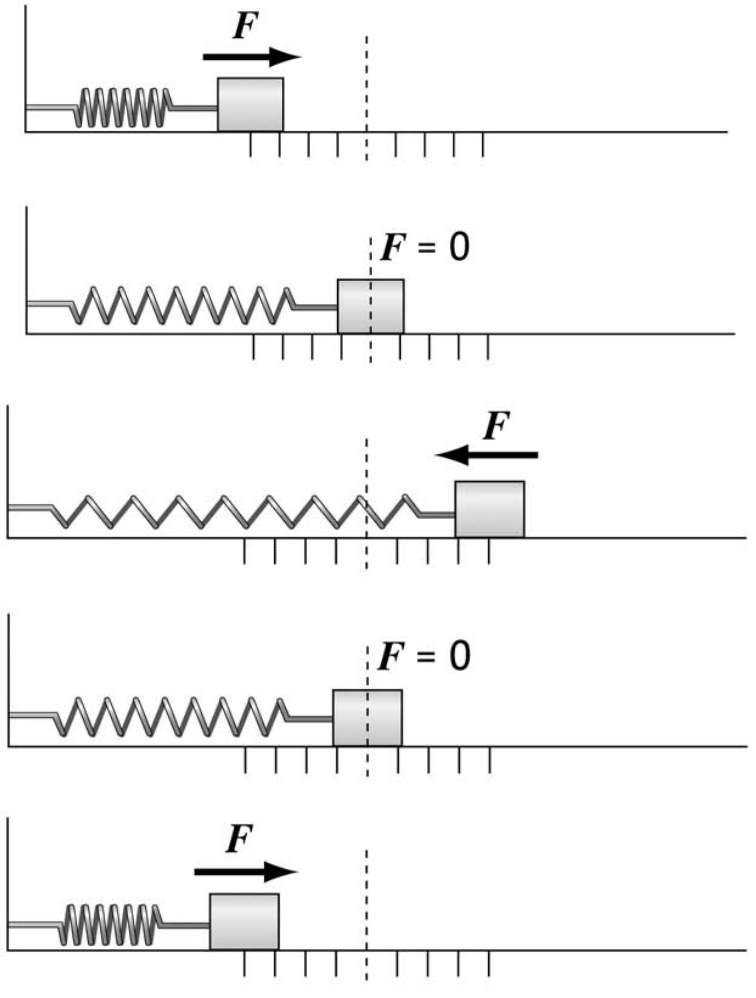
Simple Harmonic Motion

In a mass spring system, a cycle is a full to-and-fro motion (same as one trip around a circle in UCM).

Period is the time to complete one cycle.

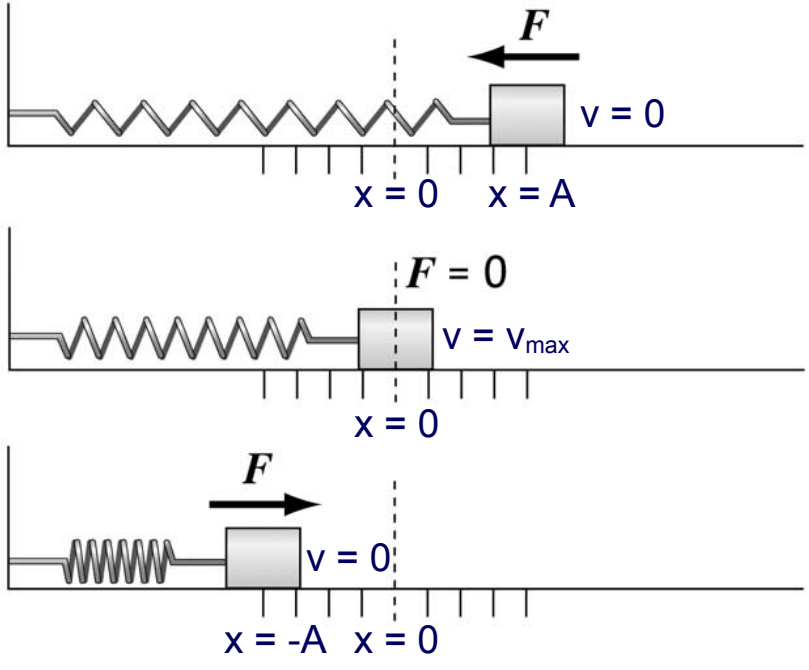
Frequency is the number of cycles completed per second.



Simple Harmonic Motion

Displacement is measured from the equilibrium point (x).

Amplitude (A) is the maximum displacement (equal to the radius in UCM).



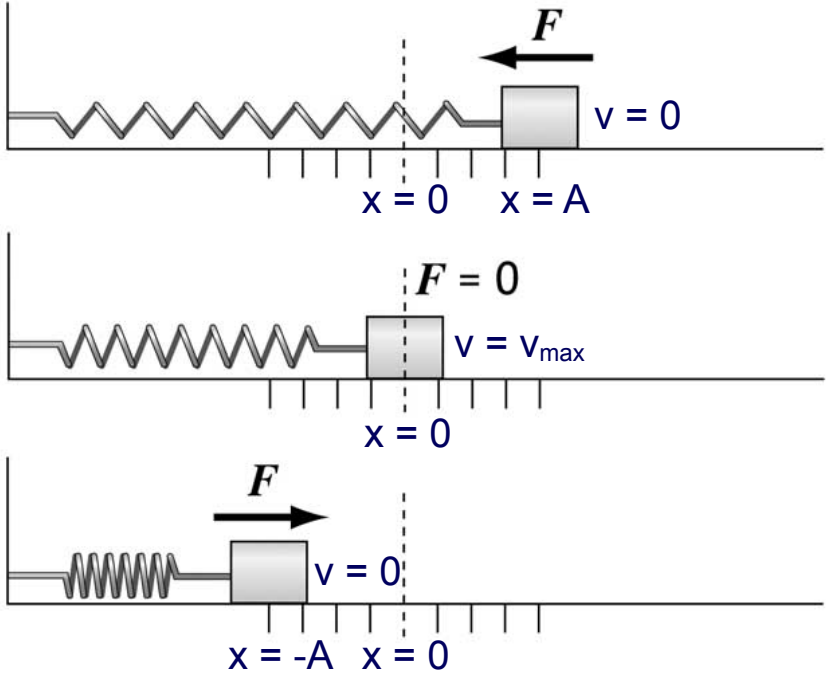
Simple Harmonic Motion

There is a point where the spring is neither stretched nor compressed; this is the equilibrium position.

We measure displacement from that point (x = 0).

The force exerted by the spring depends on the displacement:

$$\vec{F} = -k\vec{x}$$



12 A spring whose spring constant is 20N/m is stretched 0.20m from equilibrium; what is the magnitude of the force exerted by the spring?

- A 4.0 N
- B 5.0 N
- C 8.0 N
- D 9.0 N
- E I need help

Answer

https://www.njctl.org/video/?v=-CO5LHBWb4Y

13 A spring whose spring constant is 150 N/mexerts a force of 30N on the mass in a mass-spring system. How far is the mass from equilibrium?

- A 0.010 m
- B 0.050 m
- C 0.13 m
- D 0.20 m
- E I need help

Answer

14 A spring exerts a force of 50 N on the mass in a mass-spring system when it is 2.0 m from equilibrium. What is the spring's spring constant?

- ❍ 25 N/m
- ❍ 31 N/m
- ❍ 39 N/m
- ❍ 43 N/m
- ❍ I need help

Answer

Simple Harmonic Motion

$$\vec{F} = -k\vec{x}$$

The minus sign indicates that it is a restoring force – it is directed to restore the mass to its equilibrium position.

k is the spring constant

The force is not constant, so the acceleration is not constant either

https://www.njctl.org/video/?v=BOzHwXYc4Us

Simple Harmonic Motion

The maximum force exerted on the mass is when the spring is most stretched or compressed (x = -A or +A):

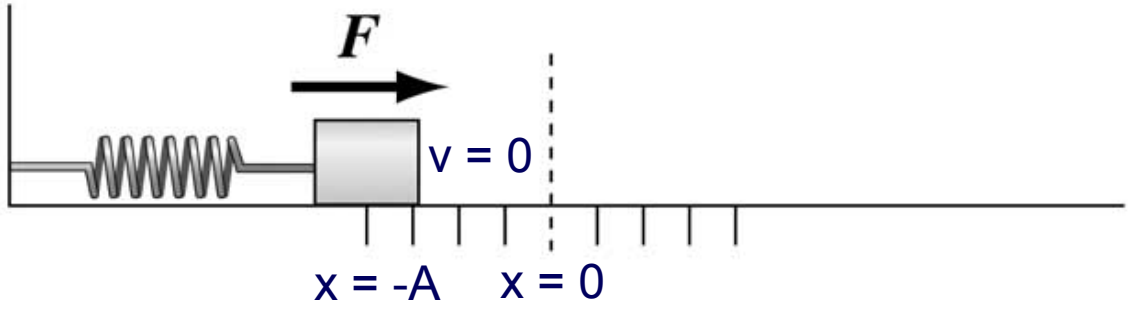
$$F = -kA \text{ (when } x = -A \text{ or } +A)$$

The minimum force exerted on the mass is when the spring is not stretched at all (x = 0)

$$F = 0 \text{ (when } x = 0)$$

Simple Harmonic Motion

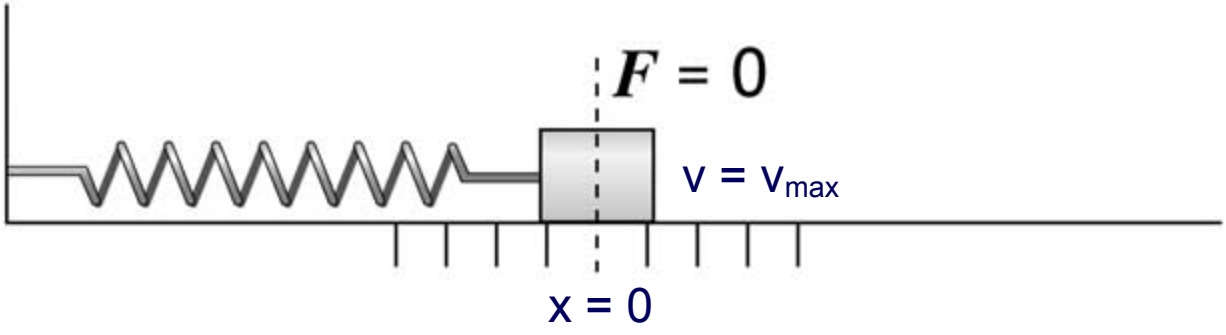
When the spring is all the way compressed:



- The displacement is at the negative amplitude.
- The force of the spring is in the positive direction.
- The acceleration is in the positive direction.
- The velocity is zero.

Simple Harmonic Motion

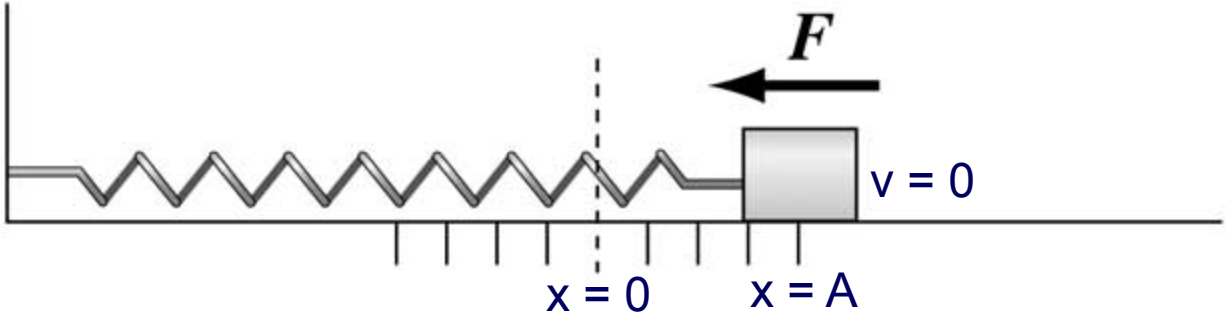
When the spring is at equilibrium and heading in the positive direction:



- The displacement is zero.
- The force of the spring is zero.
- The acceleration is zero.
- The velocity is positive and at a maximum.

Simple Harmonic Motion

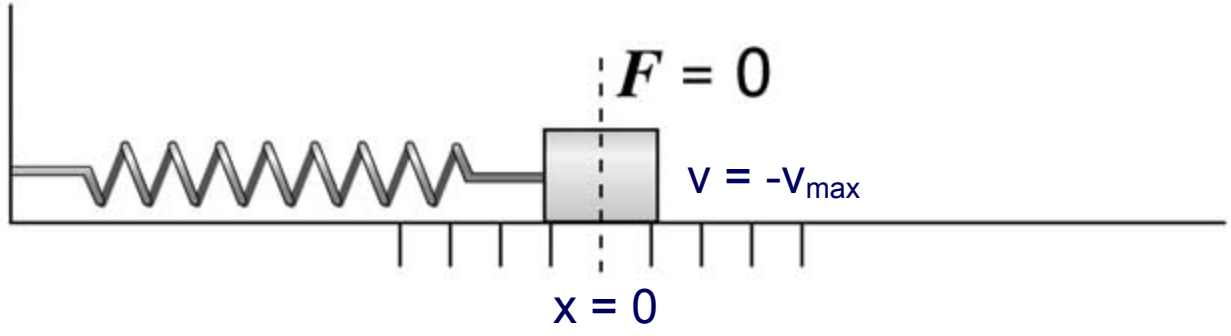
When the spring is all the way stretched:



- The displacement is at the positive amplitude.
- The force of the spring is in the negative direction.
- The acceleration is in the negative direction.
- The velocity is zero.

Simple Harmonic Motion

When the spring is at equilibrium and heading in the negative direction:

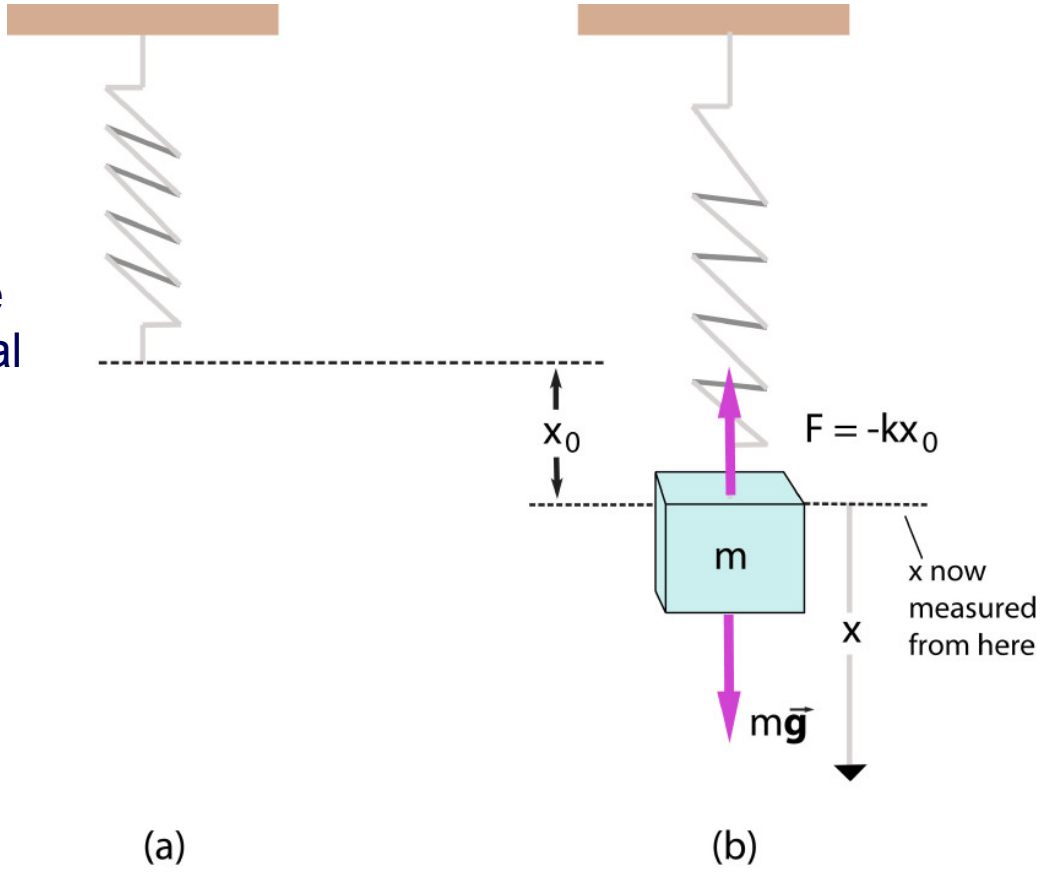


- The displacement is zero.
- The force of the spring is zero.
- The acceleration is zero.
- The velocity is negative and at a maximum.

Mass-Spring System

If the spring is hung vertically, the only change is in the equilibrium position, which is at the point where the spring force equals the gravitational force.

The effect of gravity is cancelled out by changing to this new equilibrium position.



15 At which location(s) is the magnitude of the force on the mass in a mass-spring system a maximum?

- A x = A
- B x = 0
- C x = -A
- D x = A and x = -A
- E I need help

Answer

16 At which location(s) is the magnitude of the force on the mass in a mass-spring system a minimum?

- A $x = A$
- B $x = 0$
- C $x = -A$
- D $x = A$ and $x = -A$
- E I need help

Answer

Energy and Simple Harmonic Motion

Any vibrating system where the restoring force is proportional to the negative of the displacement is in simple harmonic motion (SHM), and is often called a simple harmonic oscillator.

Also, SHM requires that a system has two forms of energy and a method that allows the energy to go back and forth between those forms.

https://www.njctl.org/video/?v=nLAXgvCjIgU

Energy in the Mass-Spring System

There are two types of energy in a mass-spring system.

The energy stored in the spring because it is stretched or compressed:

$$EPE = \frac{1}{2}kx^2$$

AND

The kinetic energy of the mass:

$$KE = \frac{1}{2}mv^2$$

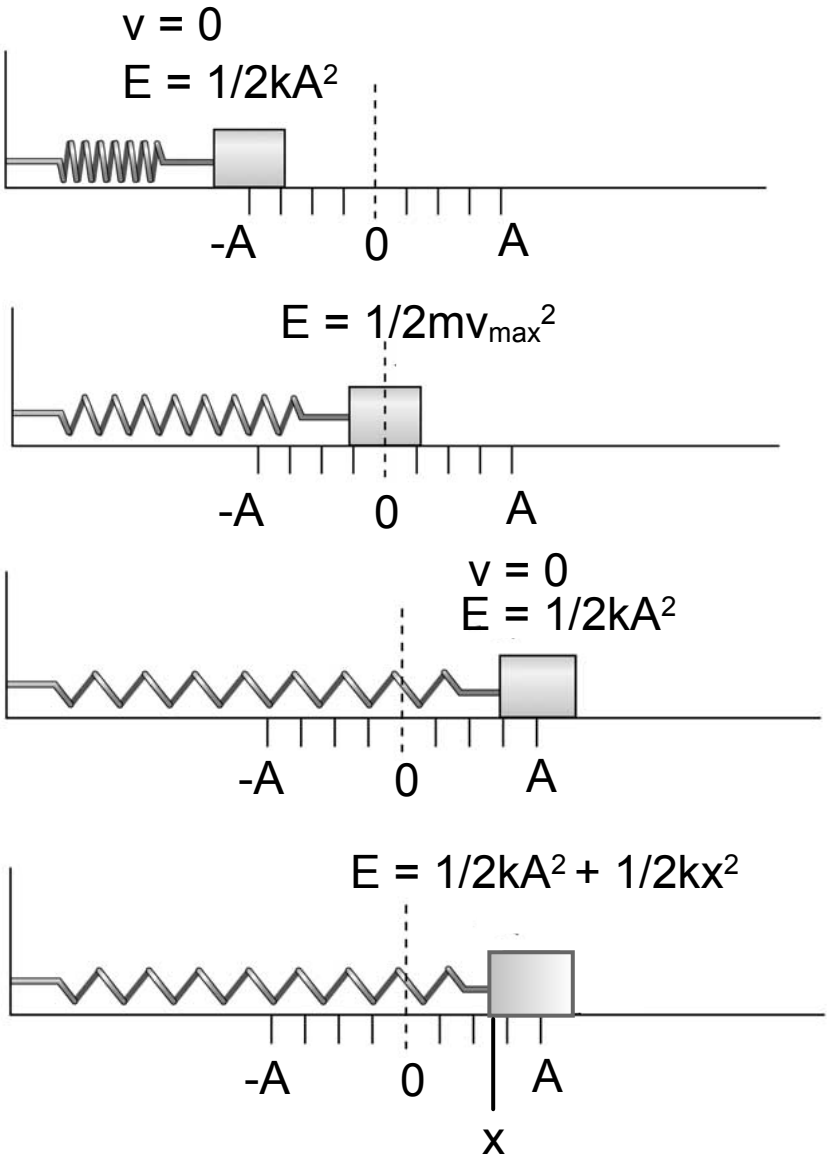
Energy in the Mass-Spring System

At any moment, the total energy of the system is constant and comprised of those two forms.

$$E_{Total} = EPE + KE$$

$$E_{Total} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

The total mechanical energy is constant.



When the mass is at the limits of its motion (x = A or x = -A), the energy is all potential:

$$E_{Total} = \frac{1}{2}kx^2$$

When the mass is at equilibrium (x = 0) the spring is not stretched and all the energy is kinetic:

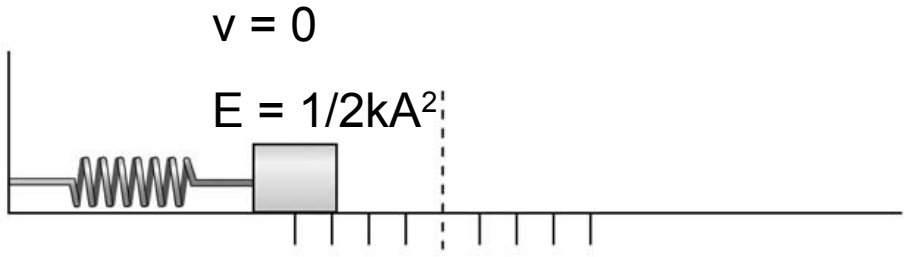
$$E_{Total} = \frac{1}{2}mv^2$$

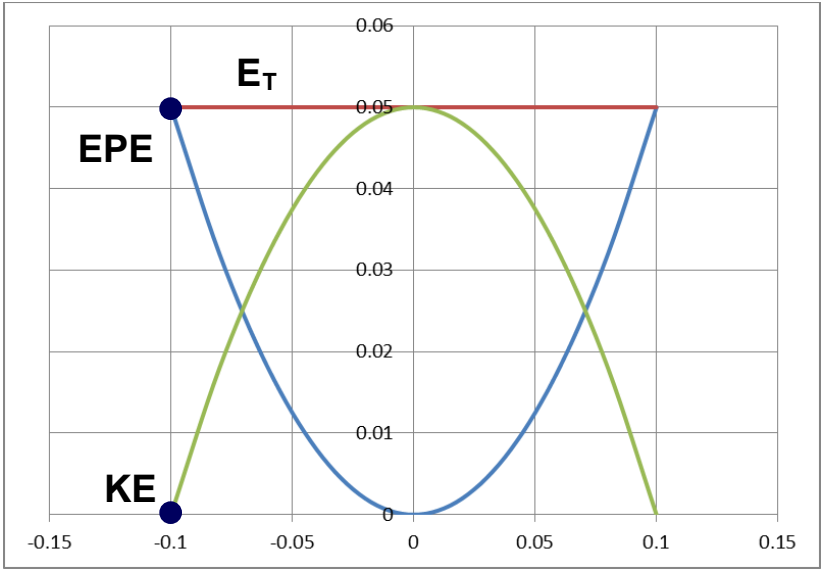
The total energy is constant:

$$E_{Total} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

Energy in the Mass-Spring System

When the spring is all the way compressed....



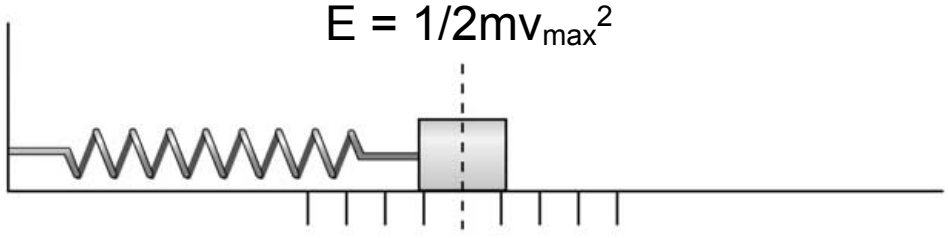


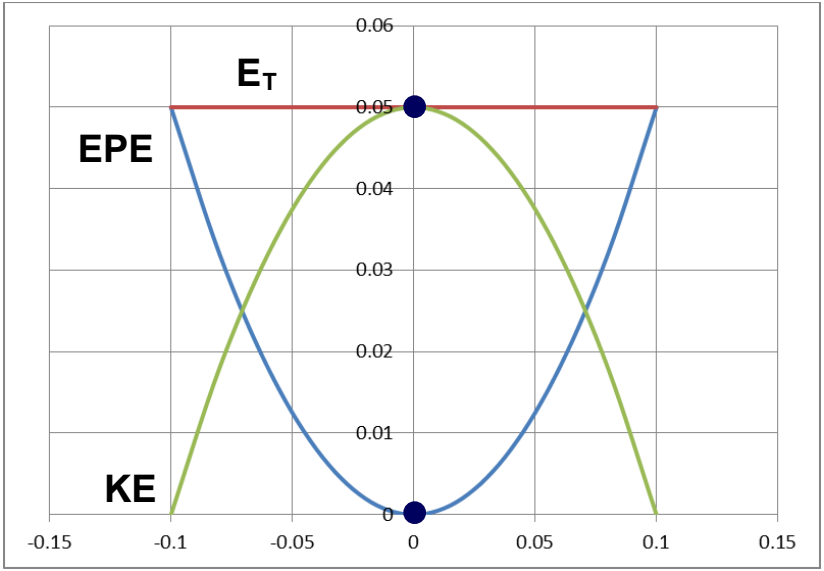
- EPE is at a maximum.
- KE is zero.
- Total energy is constant.

Energy in the Mass-Spring System

When the spring is passing through the equilibrium....

$E = 1/2mv_{max}^2$

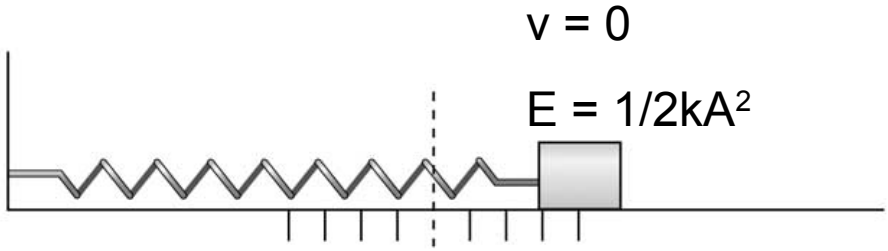


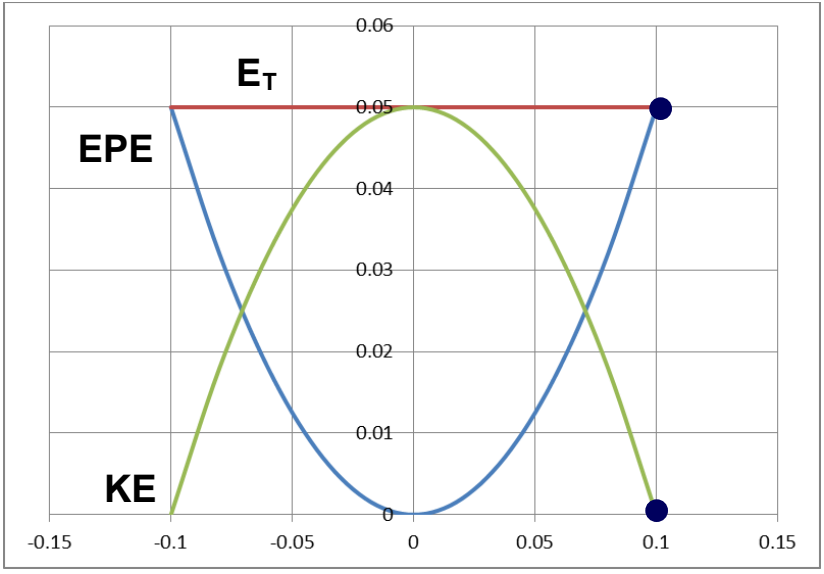


- EPE is zero.
- KE is at a maximum.
- Total energy is constant.

Energy in the Mass-Spring System

When the spring is all the way stretched....





- EPE is at a maximum.
- KE is zero.
- Total energy is constant.

17 At which location(s) is the kinetic energy of a mass-spring system a maximum?

- A $x = A$
- B $x = 0$
- C $x = -A$
- D $x = A$ and $x = -A$
- E I need help

Answer

18 At which location(s) is the spring potential energy (EPE) of a mass-spring system a maximum?

- A $x = A$
- B $x = 0$
- C $x = -A$
- D $x = A$ and $x = -A$
- E I need help

Answer

19 At which location(s) is the total energy of a mass-spring system a maximum?

- A $x = A$
- B $x = -A$
- C $x = A$ and $x = -A$
- D All of the above
- E I need help

Answer

20 At which location(s) is the kinetic energy of a mass-spring system a minimum?

- A $x = A$
- B $x = 0$
- C $x = -A$
- D $x = A$ and $x = -A$
- E I need help

Answer

Problem Solving using Energy

Since the energy is constant, and the work done on the system is zero, you can always find the velocity of the mass at any location by using

$$E_0 = E_f$$

The most general equation becomes

$$\frac{1}{2}kx_0^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}kx_f^2 + \frac{1}{2}mv_f^2$$

But usually this is simplified by being given the energy at some point where it is all EPE (x = A or -A) or when it is all KE (x = 0).

https://www.njctl.org/video/?v=QCilllv0wOE

21 What is the total energy of a mass-spring system if the mass is 2.0kg, the spring constant is 200N/m and the amplitude of oscillation is 3.0m?

- A 300 J
- B 450 J
- C 600 J
- D 900 J
- E I need help

Answer

https://www.njctl.org/video/?v=xRRoeYwMMNM

22 What is the maximum velocity of the mass in the mass-spring system from the previous slide: the mass is 2.0kg, the spring constant is 200N/m and the amplitude of oscillation is 3.0m?

- ❍ 6.0 m/s
- ❍ 15 m/s
- ❍ 30 m/s
- ❍ 40 m/s
- ❍ I need help

Answer

https://www.njctl.org/video/?v=c8UfFvgqLjE

The Period and Frequency of a Mass-Spring System

We can use the period and frequency of a particle moving in a circle to find the period and frequency:

$$KE = EPE$$

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}kx^2$$

$$mv^2 = kx^2$$

$$m\left(\frac{2\pi r}{T}\right)^2 = kx^2$$

$$T^2 = \frac{m(2\pi)^2 x^2}{kx^2} \qquad (r = x)$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

https://www.njctl.org/video/?v=sFkh-hK7D1g

Demo

Lab

23 What is the period of a mass-spring system if the mass is 4.0kg and the spring constant is 64N/m?

- A 0.39 s
- B 0.88 s
- C 1.0 s
- D 1.6 s
- E I need help

Answer

https://www.njctl.org/video/?v=U-Xb4tnhQas

24 What is the frequency of the mass-spring system from the previous slide; the mass is 4.0kg and the spring constant is 64N/m?

- A 0.41 Hz
- B 0.64 Hz
- C 0.77 Hz
- D 0.93 Hz
- E I need help

Answer

https://www.njctl.org/video/?v=baHcwbn3wmI

Simple Pendulum

Demo

https://www.njctl.org/video/?v=KQi7N2nAWDM

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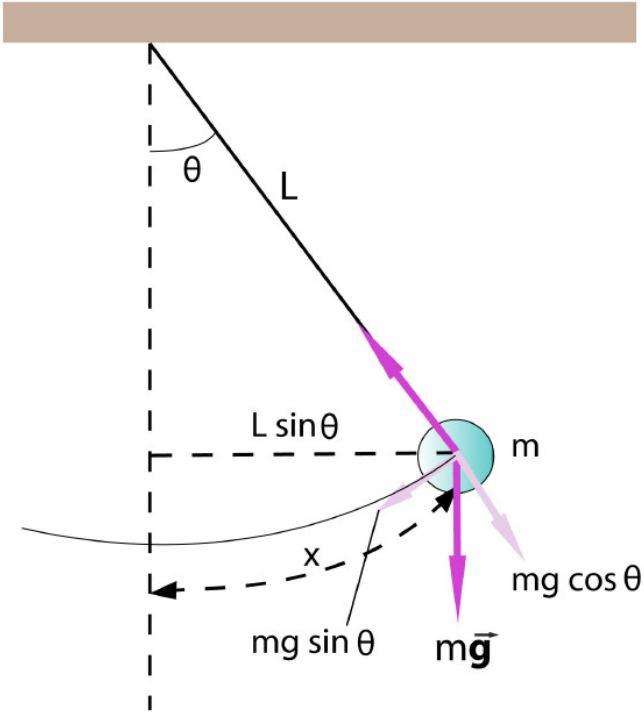
The Simple Pendulum

A simple pendulum consists of a mass at the end of a lightweight cord. We assume that the cord does not stretch, and that its mass is negligible.

*The Simple Pendulum



In order to be in SHM, the restoring force must be proportional to the negative of the displacement. Here we have:

$$F = -mg\sin\theta$$

which is proportional to sin θ and not to θ itself.

We don't really need to worry about this because for small angles (less than 15 degrees or so), sin $\theta \approx \theta$ and $x = L\theta$. So we can replace sin θ with x/L.

$$F \approx -\frac{mg}{L}x$$

https://njctl.org/video/?v=KQi7N2nAWDM

The Simple Pendulum

$F \approx -\frac{mg}{L}x$ has the form of $F = -kx$ if $k = \frac{mg}{L}$

But we learned before that $T = 2\pi\sqrt{\frac{m}{k}}$

Substituting for k $T = 2\pi\sqrt{\frac{m}{\frac{mg}{L}}}$

$T = 2\pi\sqrt{\frac{L}{g}}$ $f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$

Notice the "m" canceled out, the mass doesn't matter.

25 What is the period of a pendulum with a length of 2.0m near the surface of the earth?

- A 2.2 s
- B 2.8 s
- C 3.1 s
- D 3.8 s
- E I need help

Answer

https://www.njctl.org/video/?v=EEk97IITwzo

26 What is the frequency of the pendulum of the previous slide (a length of 2.0 m near the surface of the earth)?

- A 0.11 Hz
- B 0.20 Hz
- C 0.31 Hz
- D 0.36 Hz
- E I need help

Answer

https://www.njctl.org/video/?v=cszeokr5BIM

The Simple Pendulum

So, as long as the cord can be considered massless and the amplitude is small, the period does not depend on the mass.

https://www.njctl.org/video/?v=vfBJrvCm090

27 Which of the following factors affect the period of a pendulum?

- A the acceleration due to gravity
- B the length of the string
- C the mass of the pendulum bob
- D A & B
- E A & C
- F B & C
- G I need help

Answer

Energy in the Pendulum

The two types of energy in a pendulum are:

Gravitational Potential Energy

$$GPE = mgh$$

AND

The kinetic energy of the mass:

$$KE = \frac{1}{2}mv^2$$

https://www.njctl.org/video/?v=n1_Li_hpunU

Energy in the Pendulum

At any moment in time the total energy of the system is contant and comprised of those two forms.

$$E_{total} = GPE + KE$$

$$E_{total} = mgh + \frac{1}{2}mv^2$$

The total mechanical energy is constant.

28 What is the total energy of a 1 kg pendulum if its height, at its maximum amplitude is 0.20m above its height at equilibrium?

- A 2 J
- B 5 J
- C 9 J
- D 10 J
- E I need help

Answer

https://www.njctl.org/video/?v=5yy_pfZddpk

29 What is the maximum velocity of the pendulum's mass from the previous slide (its height at maximum amplitude is 0.20m above its height at equilibrium)?

- 0.80 m/s
- 1.1 m/s
- 1.4 m/s
- 2.0 m/s
- I need help

Answer

https://www.njctl.org/video/?v=ZboHEGGUCb0

Summary

· The period (T) is the time required for one cycle, and the frequency (f) is the number of cycles per second.

$$T = \frac{t}{n} \qquad T = \frac{1}{f}$$

$$f = \frac{n}{t} \qquad f = \frac{1}{T}$$

Summary

- For a mass on a spring:

$$E_{total} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Summary

- For a simple pendulum:

$$E_{total} = mgh + \frac{1}{2}mv^2$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$$