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AP Physics 1

Newton's Law of Universal Gravitation

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Newton's Law of Universal Gravitation

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Gravitational Force

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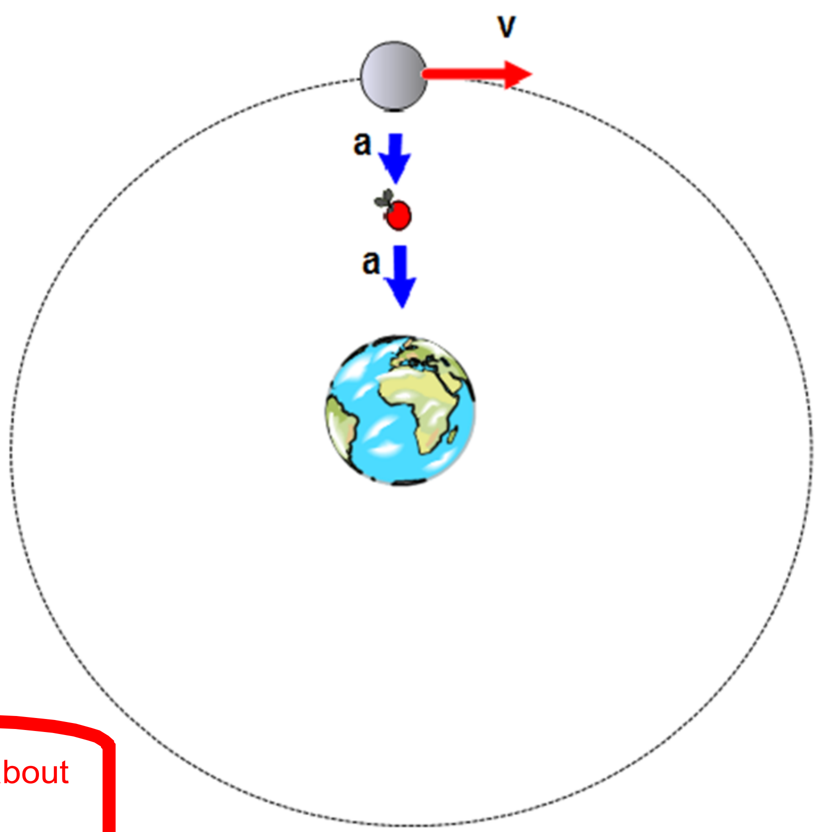
Newton's Law of Universal Gravitation

It has been well known since ancient times that Earth is a sphere and objects that are near the surface tend fall down.

Newton's Law of Universal Gravitation

Newton connected the idea that objects, like apples, fall towards the center of Earth with the idea that the moon orbits around Earth...it's also falling towards the center of Earth.

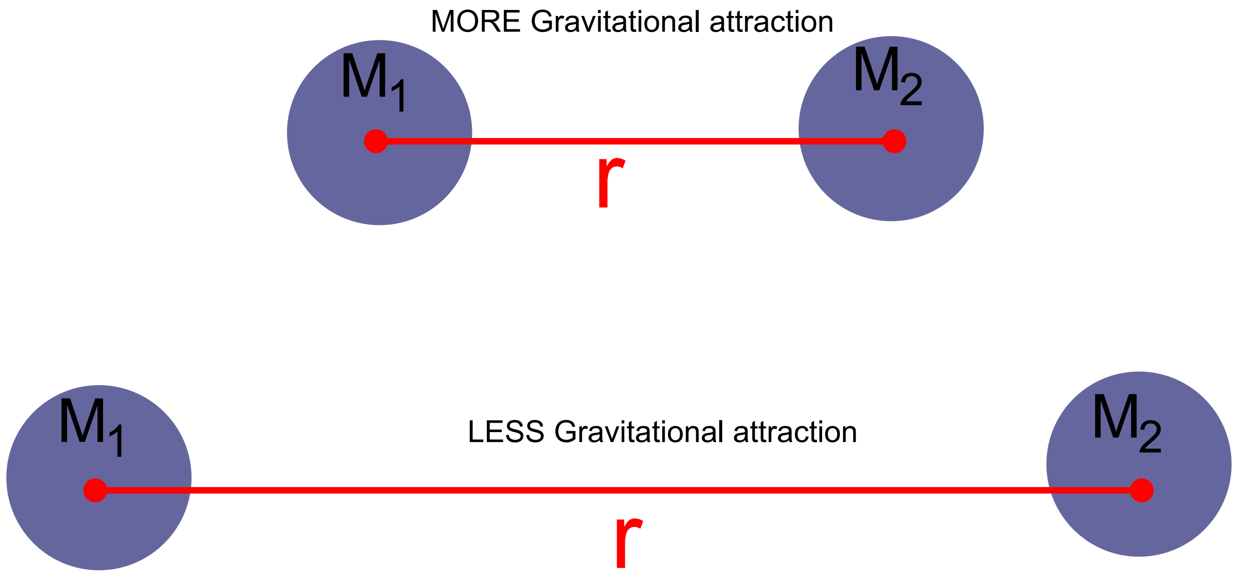
The moon just stays in circular motion since it has a velocity perpendicular to its acceleration.



click here for a cool episode of "minute physics" about why Earth orbits the sun and doesn't crash into it!

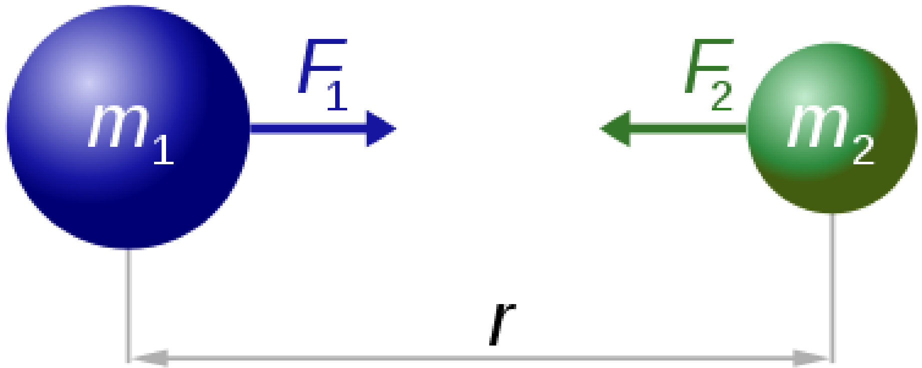
Newton's Law of Universal Gravitation

Newton concluded that all objects attract one another with a "gravitational force". The magnitude of the gravitational force decreases as <u>the centers</u> of the masses increases in distance.



Newton's Law of Universal Gravitation

Mathematically, the magnitude of the gravitational force is proportional to the masses of the objects and inversely proportional to the square of the distance between the two objects.



$$F_1 = F_2 = G\frac{m_1 \times m_2}{r^2}$$

Gravitational Constant

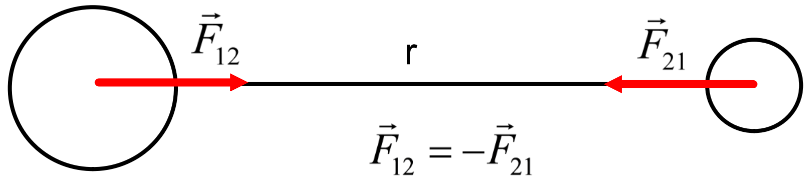
$G = 6.67 \times 10^{-11}$ $N\text{-}m^2/kg^2$

In 1798, Henry Cavendish measured G using a torsion beam balance. He did not initially set out to measure G, he was instead trying to measure the density of the Earth.

Click here for an interesting video by "Sixty Symbols" about the unusual man Henry Cavandish and his contributions to science.

Newton's Law of Universal Gravitation

The <u>direction of the force</u> is along the line connecting the centers of the two masses. Each mass feels a force of attraction towards the other mass...along that line.



Newton’s Law of Universal Gravitation

Newton's third law tells us that the force on each mass is equal.

That means that if I drop a pen, the force of Earth pulling the pen down is equal to the force of the pen pulling Earth up.

However, since the mass of Earth is so much larger, that force causes the pen to accelerate down, while the movement of Earth up is completely unmeasurable.

1 What is the magnitude of the gravitational force between Earth and its moon?

$r = 3.8 \times 10^8 m$

$m_{Earth} = 6.0 \times 10^{24} kg$

$m_{moon} = 7.3 \times 10^{22}$ kg

A 2.0×10^{18} N

B 2.0×10^{19} N

C 2.0×10^{20} N

D 2.0×10^{21} N

Answer

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

2 What is the magnitude of the gravitational force between Earth and its sun?

$r = 1.5 \times 10^{11}$ m

$m_{Earth} = 6.0 \times 10^{24}$kg

$m_{sun} = 2.0 \times 10^{30}$ kg

A 3.6×10^{-18} N

B 3.6×10^{19} N

C 3.6×10^{21} N

D 3.6×10^{22} N

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$$

Answer

3 The gravitational force between two objects is F. What is the force F' between those objects when the distance between them is halved?

A 1/2 F

B 1/4 F

C 2F

D 4F

Answer

4 The gravitational force between two objects is F. What is the force F' between those objects when the mass of one object is doubled?

A 1/4 F

B 1/2 F

C 2 F

D 4 F

Answer

5 The gravitational force between two objects is F. What is the force F' between those objects when the distance between them is doubled?

A 1/4 F

B 1/2 F

C 2 F

D 4 F

Answer

Newton's Law of Universal Gravitation

Recall that density is: $\rho = \frac{m}{V}$

Where m is mass and V is volume.

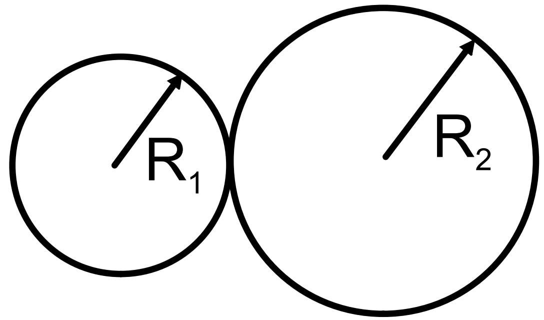
And that the volume of a sphere is: $V = \frac{4}{3}\pi r^3$

Where r is the radius of the sphere.

Now we can see what happens to the gravitational force between two objects when the mass, density, or volume is changed.

Newton's Law of Universal Gravitation

For example, lets look at the gravitational force between two spheres shown to the right.



Since $m = \rho V$ F_G can be written as:

$$F_G = \frac{G m_1 m_2}{r^2} = \frac{G(\rho_1 \frac{4}{3}\pi R_1^3)(\rho_2 \frac{4}{3}\pi R_2^3)}{(R_1 + R_2)^2}$$

6 Two solid spheres made of the same material and radii R attract each other with a gravitational force F, when they are in contact with each other. The two spheres are replaced with two new spheres of the same material with radii 2R. What is the new gravitational force between them in terms of F?

A 1/2 F

B 2 F

C 8 F

D 16 F

Answer

7 Two solid spheres made of the same material and radii R attract each other with a gravitational force F, when they are in contact with each other. One of the spheres is replaced with a new sphere of the same material with radii 3R. What is the new gravitational force between them in terms of F?

A 3/4 F

B 9/4 F

C 27/4 F

D 4/3 F

Answer

Gravitational Field

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Gravitational Field

While the force between two objects can always be computed by using the formula for F_G; it's sometimes convenient to consider one mass as creating a gravitational field and the other mass responding to that field.

$$F_G = \frac{Gm_1m_2}{r^2}$$

$$F_G = \left(\frac{Gm_1}{r^2}\right)m_2$$

$$F_G = \left(\frac{GM}{r^2}\right)m$$

$$F_G = weight = mg$$

$$\boxed{g = \frac{GM}{r^2}}$$

Gravitational Field

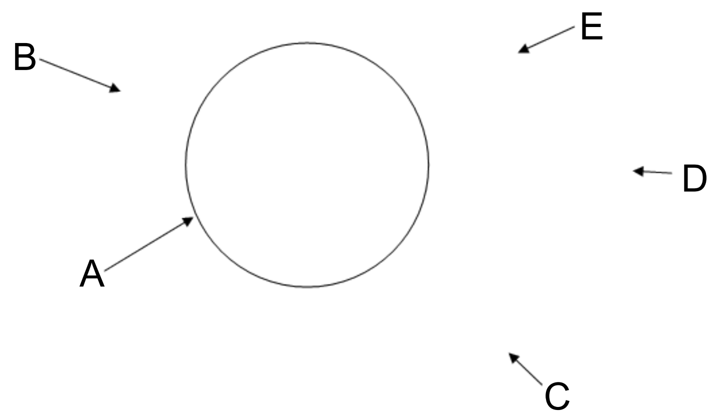
The magnitude of the gravitational field created by an object varies from location to location in space; it depends on the distance from the object and the object's mass.

Gravitational field, g, is a vector. It's direction is always towards the object creating the field.

That's the direction of the force that a test mass would experience if placed at that location. In fact, g is the acceleration that a mass would experience if placed at that location in space.

$$g = \frac{GM}{r^2}$$

8 Where is the gravitational field the strongest?



Answer

9 What happens to the gravitational field if the distance from the center of an object doubles?

A It doubles

B It quadruples

C It is cut to one half

D It is cut to one fourth

Answer

10 What happens to the gravitational field if the mass of an object doubles?

A It doubles
B It quadruples
C It is cut to one half
D It is cut to one fourth

Answer

Surface Gravity

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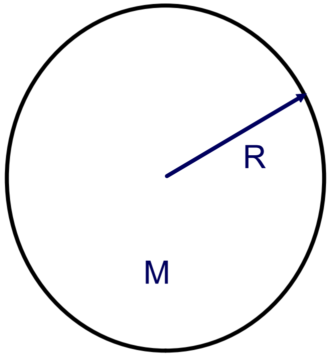
Surface Gravity

Planets, stars, moons, all have a gravitational field...since they all have mass.

That field is largest at the object's surface, where the distance from the center of the object is the smallest...when "r" is the radius of the object.

By the way, only the mass of the planet that's closer to the center of the planet than you are contributes to its gravitational field. So the field actually gets smaller if you tunnel down below the surface.



$$g = \frac{GM}{R^2}$$

11 Determine the surface gravity of Earth's moon. Its mass is 7.4×10^{22} kg and its radius is 1.7×10^{6} m.

Answer

12 Compute g for the surface of a planet whose radius is double that of the Earth and whose mass is triple that of Earth.

Answer

Surface Gravity

Again density is: $\rho = \dfrac{m}{V}$ So $m = \rho V$.

And that the volume of a sphere is: $V = \dfrac{4}{3}\pi r^3$

Now we can see what happens to the surface gravity of a planet when the mass, density, or volume is changed.

Surface Gravity

For example, we can rewrite the equation for surface gravity in terms of density and radius.

$$g = \frac{GM}{R^2} = \frac{G\left(\rho \frac{4}{3}\pi R^3\right)}{R^2} = \frac{4}{3} G\pi\rho R$$

13 Compute g for the surface of a planet whose radiusis double that of the Earth and whose density is the same as that of Earth.

A $1/4\ g_{earth}$
B $1/2\ g_{earth}$
C $2\ g_{earth}$
D $4\ g_{earth}$

Answer

14 Compute g for the surface of a planet whose radiusis the same as that of the Earth and whose density is 1/3 that of Earth.

A 1/9 g_{earth}
B 1/3 g_{earth}
C 3 g_{earth}
D 9 g_{earth}

Answer

15 Compute g for the surface of a planet whose radius is half that of Earth and whose density is 3/2 that of Earth.

A 1.7 N/kg
B 2.5 N/kg
C 7.4 N/kg
D 13 N/kg

Answer

Gravitational Field in Space

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Gravitational field in space

While gravity gets weaker as you get farther from a planet, it never becomes zero.

There is always some gravitational field present due to every planet, star and moon in the universe.

Gravitational field in space

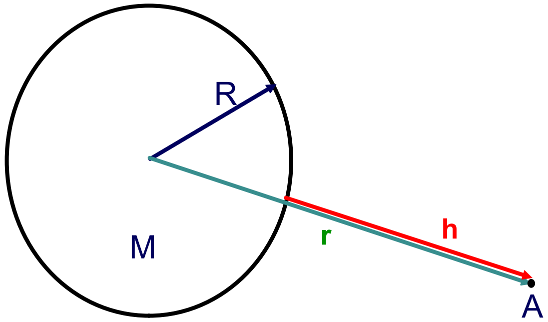
The local gravitational field is usually dominated by nearby masses since gravity gets weaker as the inverse square of the distance.

The contribution of a planet to the local gravitational field can be calculated using the same equation we've been using. You just have to be careful about "r".

Gravitational field in space

The contribution of a planet to the local gravitational field can be calculated using the same equation we've been using. You just have to be careful about "r".

If a location, "A", is a height "h" above a planet of radius "R", it is a distance "r" from the planet's center, where r = R + h.



$$g_A = \frac{GM}{r^2}$$

$$g_A = \frac{GM}{(R+h)^2}$$

16 Determine the gravitational field of Earth at a height of 6.4×10^6 m (1 Earth radius). Earth's mass is 6.0×10^{24} kg and its radius is 6.4×10^6 m.

Answer

17 Determine the gravitational field of Earth at a height 2.88×10^8 m above its surface (the height of the moon above Earth).

Earth's mass is 6.0×10^{24} kg and its radius is 6.4×10^6 m.

Answer

The International Space Station (ISS)

The International Space Station (ISS) is a research facility that is in a Low Earth Orbit and can be seen from Earth with the naked eye! Its on-orbit construction began in 1998.

It orbits at an altitude of approximately 350 km (190 mi) above the surface of the Earth, and travels at an average speed of 27,700 kilometers (17,210 mi) per hour. This means the astronauts see 15 sunrises everyday!

18 The occupants of the space station appear to be weightless, they float. Determine the strength of Earth's gravitational field acting on astronaut's in the international space station.

Earth's mass is 6.0×10^{24} kg and its radius is 6.4×10^{6} m. The ISS is 350km (3.5×10^{5}m) above the surface of the earth.

Answer

19 How does the gravitational field acting on the occupants in the space station compare to that acting on you now.

A It's the same.

B It's slightly less.

C It's about half as strong.

D There is no gravity acting on them.

Answer

Orbital Motion

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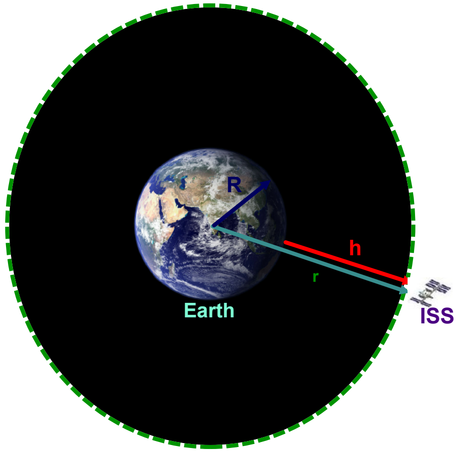
Orbital Motion

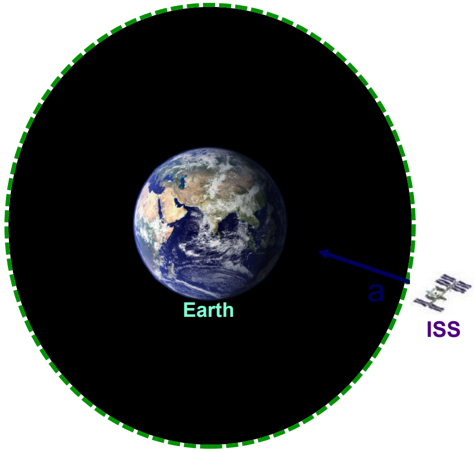


We've already determined that the gravitational field acting on the occupants of the space station, and on the space station itself, is not very different than the field acting on us here on Earth's surface.

How come they don't fall to Earth?

This diagram should look really familiar….

Orbital Motion

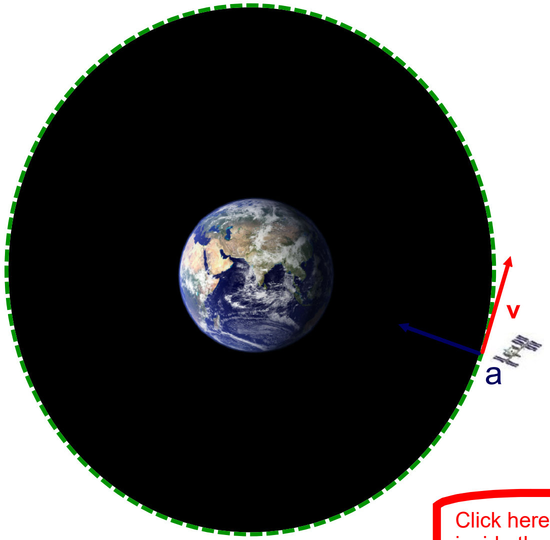


The gravitational field will be pointed towards the center of Earth and represents the acceleration that a mass would experience at that location (regardless of the mass).

In this case any object would simply fall to Earth.

How could that be avoided?

Orbital Motion



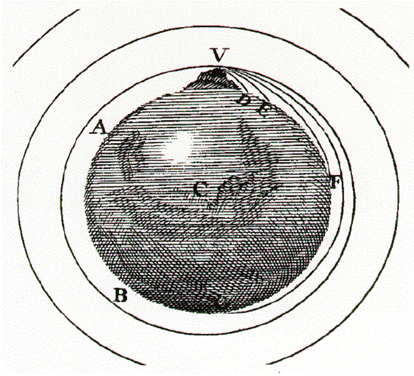
If the object has a tangential velocity perpendicular to its acceleration, it will go in a circle.

It will keep falling to Earth, but never strike Earth.

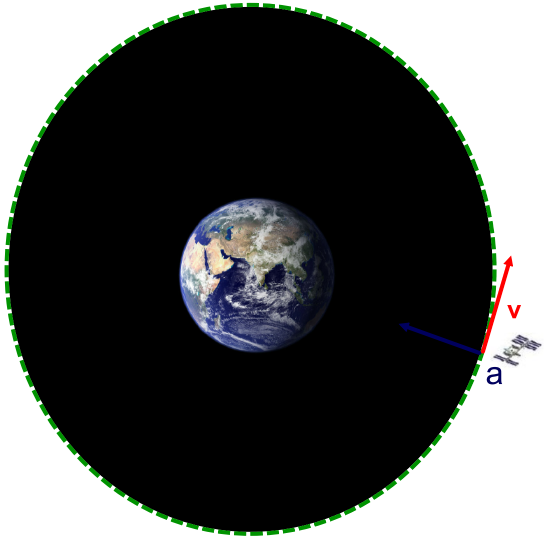
Click here for an interesting look at why the astronauts inside the space station appear to be weightless.

Orbital Motion

Here is Newton's own drawing of a thought experiment where a cannon on a very high mountain (above the atmosphere) shoots a shell with increasing speed, shown by trajectories for the shell of D, E, F, and G and finally so fast that it never falls to earth, but goes into orbit.



click here for another look at trajectories and orbital motion by Kahn Academy



Orbital Motion

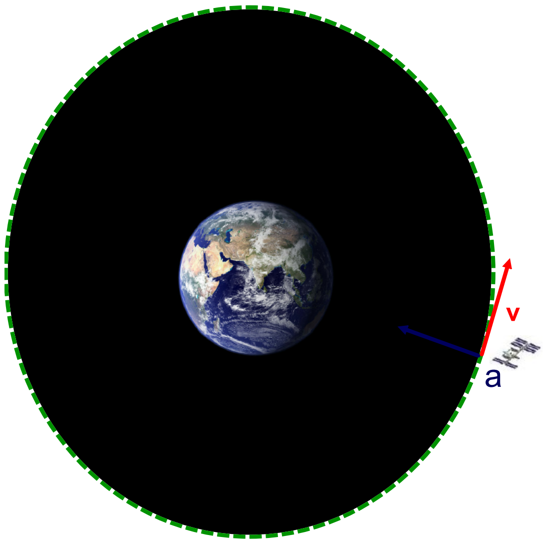
We can calculate the velocity necessary to maintain a stable orbit at a distance "r" from the center of a planet of mass "M".

$$\Sigma F = ma$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{GM}{r} = v^2$$

$$v = \sqrt{\frac{GM}{r}}$$



Orbital Motion

From that, we can calculate the period, T, of any object's orbit.

$$v = \sqrt{\frac{GM}{r}} \quad \text{or} \quad v = \sqrt{gr}$$

$$v = \frac{2\pi r}{T}$$

$$T = \frac{2\pi r}{v}$$

20 Compute g at a distance of 7.3×10^{8} m from the center of a spherical object whose mass is 3.0×10^{27} kg.

Answer

21 Use your previous answer to determine the velocity, both magnitude and direction, for an object orbiting at a distance of 7.3×10^8 m from the center of a spherical object whose mass is 3.0×10^{27} kg.

Answer

22 Use your previous answer to determine the orbital period of for an object orbiting at a distance of 7.3×10^{8} m from the center of a spherical object whose mass is 3.0×10^{27} kg.

Answer

23 Compute g at a height of 59 earth radii above the surface of Earth.

Answer

24 Use your previous answer to determine the velocity, both magnitude and direction, for an object orbiting at height of 59 R_E above the surface of Earth.

Answer

25 Use your previous answer to determine the orbital period of an object orbiting at height of 59 R_E above the surface of Earth.

Answer

26 Two satellites orbit two different planets with the same radius but different densities. The density of the first planet is ρ and the density of the second planet is 2ρ. Both satellites are at the same height above their planets. What is the orbital speed of the second satellite in terms if the orbital speed of the first?

A $v_2 = \sqrt{(2)}v_1$
B $v_2 = 2v_1$
C $v_2 = 4v_1$
D $v_2 = v_1/2$

Answer

27 Two satellites orbit two different planets at the same distance to the center of the planet and with the same density but different radii. The radius of the first planet is R and the radius of the second planet is 2R. What is the orbital speed of the second satellite in terms if the orbital speed of the first?

A $v_2 = \sqrt{(2)}v_1$
B $v_2 = 2v_1$
C $v_2 = v_1$
D $v_2 = 2\sqrt{(2)}v_1$

Answer

Kepler's Third Law of Motion

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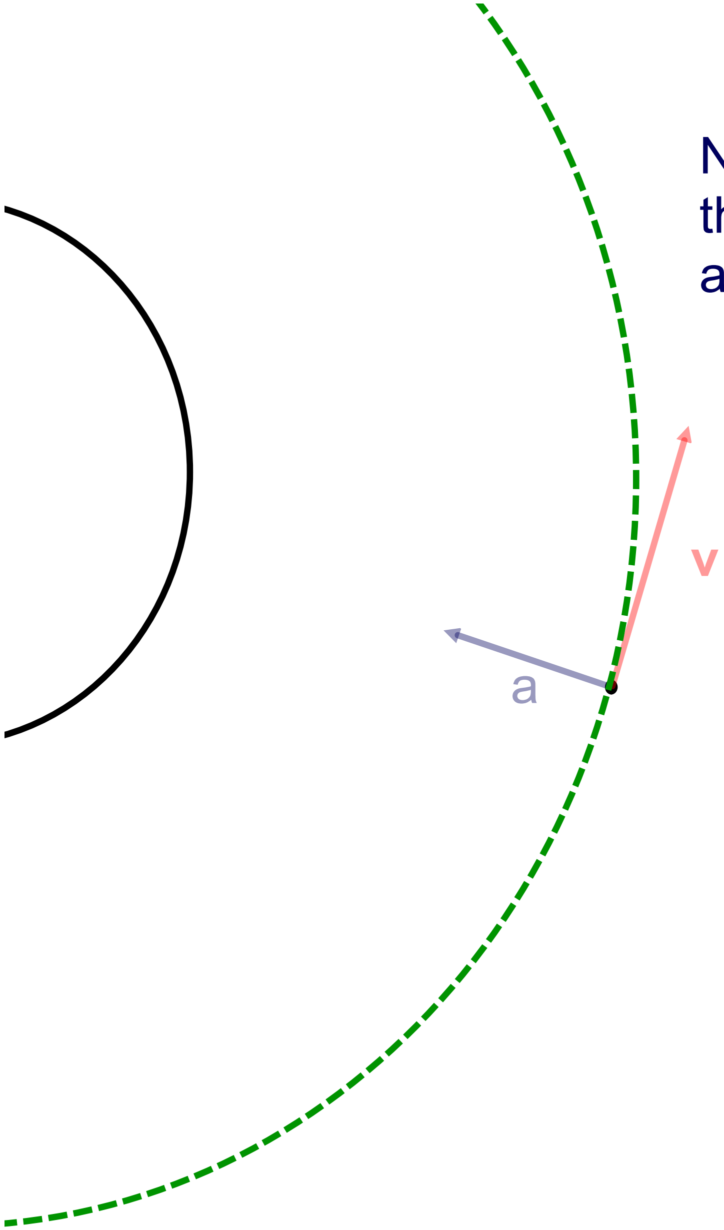


Orbital Motion

Now, we can find the relationship between the period, T, and the orbital radius, r, for any orbit.

$$v = \frac{2\pi r}{T}$$

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

$$\frac{GM}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$GMT^2 = 4\pi^2 r^3$$

$$\boxed{\frac{T^2}{r^3} = \frac{4\pi^2}{GM}}$$

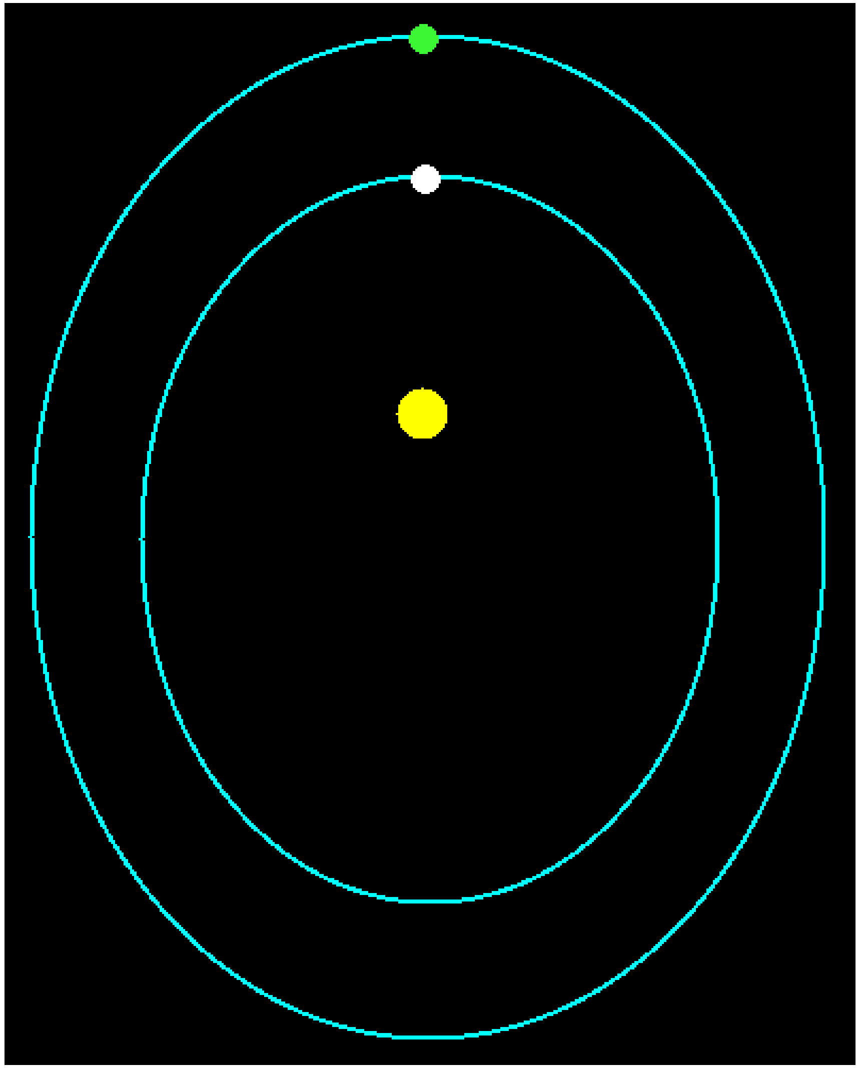
Kepler's Third Law

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

Kepler had noted that the ratio of T^2 / r^3 yields the same result for all the planets. That is, the square of the period of any planet's orbit divided by the cube of its distance from the sun always yields the same number.

We have now shown why: $(4\pi^2)$ / (GM) is a constant; its the same for all orbiting objects, where M is the mass of the object being orbited; it is independent of the object that is orbiting.

Kepler's Third Law



If you know the period (T) of a planet's orbit, you can determine its distance (r) from the sun.

Since all planets orbiting the sun have the same period to distance ratio, the following is true:

$$\frac{T^2}{r^3} = \frac{4\pi^2}{GM}$$

$$\frac{T_{(white)}^2}{r_{(white)}^3} = \frac{T_{(green)}^2}{r_{(green)}^3}$$

28 The period of the Moon is 27.3 days and its orbital radius is 3.8 x 10^8 m. What would be the orbital radius of an object orbiting Earth with a period of 20 days?

Answer

29 What is the orbital period (in days) of an unknown object orbiting the sun with an orbital radius of twice that of earth?

Answer